APPLIED ASPECTS OF OPTIMIZATION OF INVESTMENT PORTFOLIOS OF UNIT INVESTMENT FUNDS

(on the example of the activity of joint-stock company „Sberbank Asset Management“)

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Annotation: *Mutual investment fund „Sberbank Asset management“ according to the rating of Mutual Funds on profitability takes a leading position. Under the management of this company there are four funds: the fund, shares, the MICEX index, and fund „Russian oil“. Funds are managed in accordance with the requirements of Russian legislation to investment funds, the composition and structure of their assets and other requirements. At the same time, the current Russian legislation in the field of investment funds often does not take into account the problems of risk management when forming optimal portfolios in terms of the acceptable ratio of profitability and risk, which leads to lost profits or financial losses of the shareholders of such funds. For the optimal portfolio, you can use the methods of regression analysis and linear programming. In the first case, the study examined the dependence of the variables X and Y, for which the estimated and estimated value of the investment share was taken. In the second case, the investment portfolio was optimized in order to reduce its risk level at a given level of expected returns on two parameters: covariance and diversification of its financial assets using elements of linear programming. Based on the results of the regression analysis, regression equations were obtained for the projected and estimated value of the unit in relation to the retrospective data of four funds: „Balanced“, „Shares“, „Russian Oil“ and „MICEX Index“. This allowed us to calculate the share of each fund in the portfolio, the profitability and risk of the portfolio, using EXCEL. At the same time, the forecast value for 5 years ahead was considered as the value of profitability, and the standard deviation of the yields of assets as a risk. The results of the calculations were as follows: the non-optimization portfolio received has a return of 55.39% per annum with a risk level of 4.83%. The portfolio can be optimized using linear programming methods. The Markovets model was used as an optimizing one, using the built-in Lagrange function. The results of calculations showed that the level decreases 55,39-49,44 = 5,95% and the level of risk is reduced by 4,83-3,17 = 1,66%. The received result of calculations can be considered, as quite satisfactory, as the general level of risk on a portfolio has decreased.*

Keywords: *regression, profitability, risk, investment fund, assets, management, investment share, optimization.*
American Economic Association: JEL Codes:
C1 Econometric and Statistical Methods and Methodology: General
C58 Financial Econometrics
G11 Portfolio Choice • Investment Decisions

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[1] *The authors' participation is as follows: Dimov, S.Hr.: from Annotation to Set of recommendations for transactions, incl., and point III. Return of the optimized portfolio; Rebelsky N.M.: point I. Mutual investment fund of Sberbank Asset management; Smirnov V.V.: point II. Annual return of the portfolio and List of used literature.*

Introduction: The subject of the study is the composition and structure of the assets of four mutual investment funds managed by the joint-stock company Sberbank Asset Management. The aim of the research is to find the optimal structure of the investment portfolio, which includes the financial assets of four funds that minimize the risk of the portfolio, given its expected return.

Materials and methods: The dynamics of the value of investment units of four mutual investment funds was measured monthly, starting from 06.02.2015. to 06.12.2017. Based on the historical data for this period, four regression equations were constructed that establish the relationship between the projected unit value and its estimated value for the period from 2015 until 2022. Using the calculated data of the four regression equations, the parameters of the initial portfolio (non-optimization) were determined. A dispersion-covariance analysis tool was used to calculate: the share of each of the four funds in the total portfolio value, the initial levels of its risk and the expected return. Next, based on the methods of linear programming, a model was constructed for the formation of an optimal investment portfolio. Based on the results of the calculations, a new portfolio structure was obtained, which reduced its risk and somewhat reduced its expected return. However, the obtained result can be considered satisfactory, since the overall risk of the portfolio has decreased.

Results: As a result of the conducted research and obtained calculations with a certain probability, it can be argued that even if the rigid Markovtz optimization conditions are not met (the normality of the profit distribution or the utility function of the investor should be quadratic), a deliberate change in the portfolio structure that reduces its overall risk is possible.

Discussion: If the initial data strongly do not correspond to at least one of the conditions for using the Markovets models, for example, the normality of the distribution of profitability, then the main parameters of this model should be adjusted taking into account the crookedness and excesses of the curve of the normal distribution of the yield of the portfolio under study.

Implications: By evaluating the risk of a specific asset from an investment portfolio, you can either consider this asset in isolation from other assets, or consider it an integral part of the portfolio. An asset that has a high level of risk when it is considered in isolation can be practically risk-free from a portfolio position and with a certain combination of assets in that portfolio.

Set of recommendations for transactions: These methods can be reduced to a set of recommendations for transactions with the portfolio, which are as follows:

1) When analyzing the appropriateness of operations with a portfolio of securities, three tasks are posed: to achieve the highest possible yield, to obtain the minimum possible risk, to obtain an acceptable value for the combination „yield / risk".

2) Yield of the portfolio is determined by a weighted average formula, so the combination of high-yield financial assets in the portfolio ensures a high return on the portfolio.

3) Adding a risk-free asset to the portfolio reduces the portfolio's profitability, while the risk of the portfolio decreases in direct proportion to the share of that asset.

4) The combination of risk assets in a portfolio can lead to a reduction in risk compared to the possession of each of these assets individually, but the outcome depends not only on the risk of the assets being pooled, but also on the nature of the relationship between their returns.

5) If the return on the asset planned for inclusion in the portfolio changes unilaterally with its yield, the risk of the new combination may change in any direction in comparison with the risk of the initial portfolio.

6) If an asset is added to the portfolio, the profitability of which varies in different ways with the portfolio return, then the risk of the new combination is usually reduced.

7) If there are a choice of two assets with the same characteristics, however, the yield of one changes unilaterally, and the second one's yield is differently directed to the portfolio return, then from the position of minimizing the risk for inclusion in the portfolio, the second asset should be preferred.

Summarizing the foregoing, we note that, while maintaining market trends, it is advisable to invest with the same structural distribution of assets that was in the formation of the portfolio. If certain negative trends appear on the market, then in this case it is recommended to restructure the portfolio using the methods described above separately or in combination. Recommendations for improving the management of the restructuring of the portfolio of financial investments are reduced, first, to the rationale for choosing the person responsible for portfolio management. And if at the first stage (when forming the portfolio) the fund manager plays a priority role in this process, then already in the medium and, especially, long-term periods, the emphasis in management should be shifted in favor of a full-time financial manager (analyst). ***And, finally, it should be recommended to increase the accuracy of forecasts - at medium terms this becomes possible***.

I. Mutual investment fund of Sberbank Asset management

Mutual investment fund of Sberbank Asset management according to the rating of Mutual Funds on profitability takes a leading position. (Ratings of mutual funds from Investfonds, http://pif.investfunds.ru/funds/rate.phtml/ 16-11-2018)

Under the management of this company are four funds: a balanced fund, a stock fund, the MICEX Index Fund, the Russian Oil Fund. Funds are managed in accordance with the requirements of Russian legislation to investment funds, the composition and structure of their assets and other requirements. At the same time, the current Russian legislation in the field of investment funds often does not take into account the problems of risk management when forming optimal portfolios in terms of the acceptable ratio of profitability and risk, which leads to lost profits or financial losses of the shareholders of such funds. To compile the optimal portfolio, you can use the methods of regression analysis and linear programming. In the first case, for the study, consider the zavimost of the variables X and Y, for which the estimated and forecasted values of the investment share are taken. Let us compose the regression equations for each fund. To determine the dependence of Y on X, it is necessary to calculate the parameters of the following linear function:

$$\hat{y}_x = a + b * x \tag{1}$$

Next, we need to solve the system of normal equations with respect to the parameters a and b:

$$\begin{cases} n * a + b\sum x = \sum y \\ a\sum x + b\sum x^2 = \sum y * x \end{cases} \tag{2}$$

Next, we construct a table of calculated data for estimating the linear regression as shown in Figure 1.

	A	B	C	D	E	F	G	H
1	№ п/п	x	y	xy	x2	y2	$\widehat{y_x}$	$\left\lvert\frac{y-\widehat{y_x}}{y}\right\rvert$

Fig. 1. Calculated data for estimating linear regression

The average value is determined by the formula:

$$\bar{x} = \frac{\sum x}{n} \qquad (3)$$

The average quadratic deviation is calculated by the formula:

$$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} \qquad (4)$$

By squaring the resulting value, we obtain the variance:

$$\sigma^2 = \frac{\sum(x-\bar{x})^2}{n} \qquad (5)$$

The parameters of the equation can also be determined from the formulas:

$$b = \frac{\frac{\sum y * x}{n} - \bar{y} * \bar{x}}{\sigma_x^2} \qquad (6)$$

$$a = \bar{y} - b * \bar{x} \qquad (7)$$

Thus, the regression equations for each fund have the form shown in Figures 2-6.

Next, we determine the linear coefficient of pair correlation by the formula:

$$r_{xy} = b\frac{\sigma_x}{\sigma_y} \qquad (8)$$

Thus, the relationship between the dependent random variable and the independent random variable is determined and how close it is.

Determine the coefficient of determination, raising the coefficient of pair correlation in a square: formula:

$$r_{xy}^2 := \left(r_{xy} = b\frac{\sigma_x}{\sigma_y} \right)^2 \qquad (9)$$

Consider an increase in the value of a share in a balanced fund. These data are shown in Figure 2. At the same time, in Figure 2 the parameters of the linear regression equation approximating the obtained series of dynamics are immediately derived.

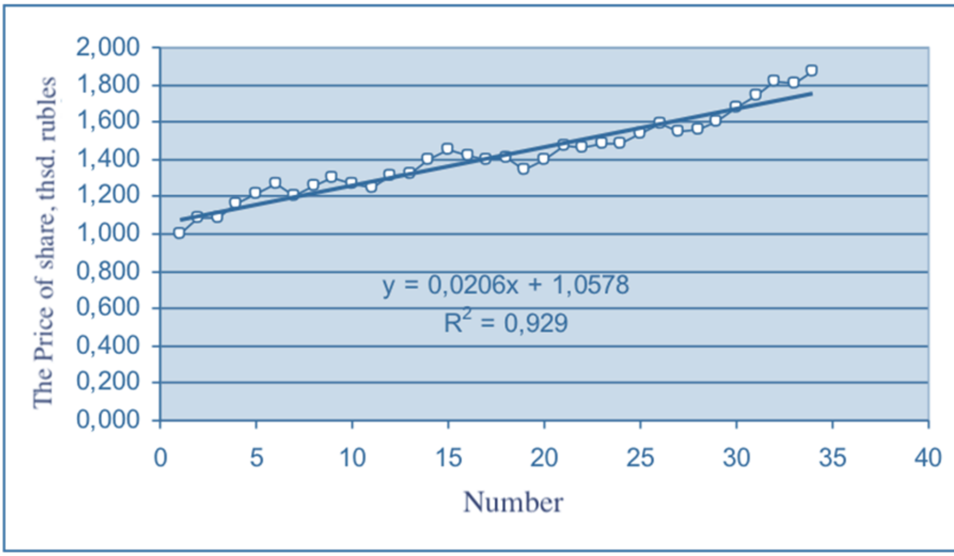

Fig. 2. Increase in the value of a unit of a balanced fund

Consider an increase in the value of a share in the shares of a stock fund. These data are shown in Figure 3. At the same time, in Figure 3 the parameters of the linear regression equation approximating the obtained series of dynamics are immediately derived.

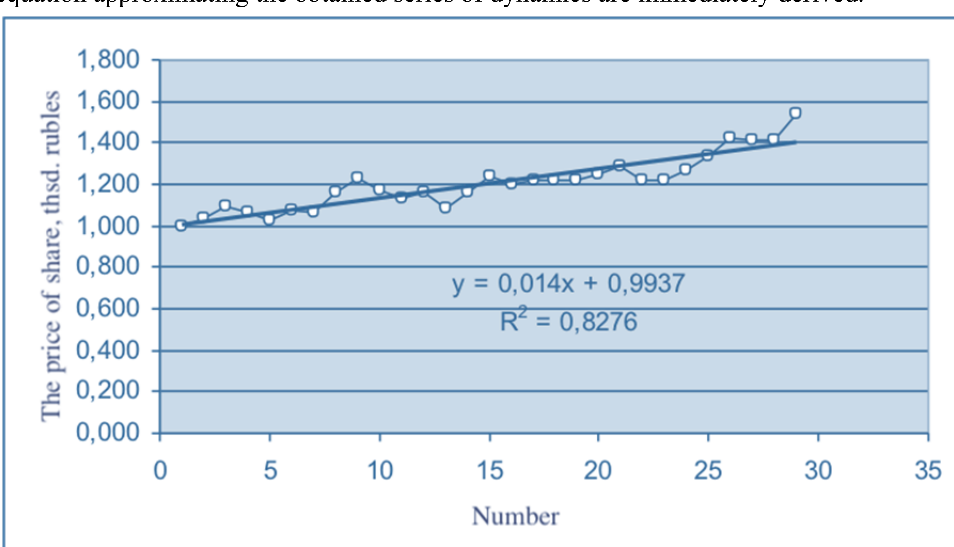

Fig. 3. Increase in the value of the share fund unit

Consider the increase in the value of a share in the shares of the Russian Oil Fund. These data are shown in Figure 4. At the same time, in Figure 4 the parameters of the linear regression equation approximating the obtained series of dynamics are immediately derived.

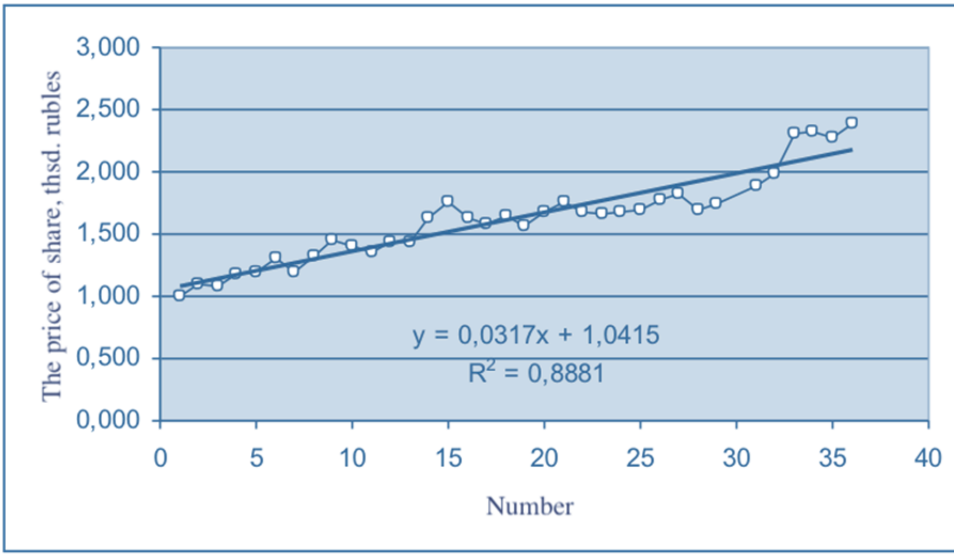

Fig. 4. Increase in the value of the fund unit „Russian Oil"

Consider an increase in the value of a share in the shares of the MICEX Index Fund. This and the data are shown in Figure 6. In this case, in Figure 6, the parameters of the linear regression equation approximating the resulting series of dynamics are immediately derived.

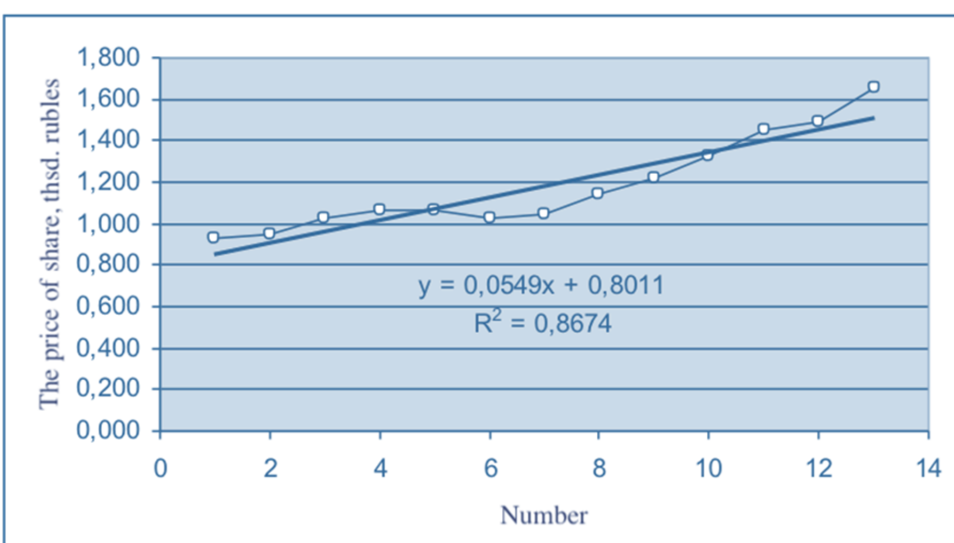

Fig. 6. Growth in the value of the fund unit „MICEX Index"

On the basis of the data obtained, we shall compile, based on the regression equations, the forecast for the growth of the value of the units for the next five years. Calculations are conducted in Excel and put them in special tables for each fund. Next, you need to calculate the share of each fund in the portfolio, based on the maximization of profits while maintaining a reasonable level of risk. Issues related to the distribution of the share of assets in the investment portfolio are considered by a number of authors, offering the following recommendations: the portfolio should consist of several assets; mathematics of the formation allows for various algorithms based on the expected return and risk calculation in the form of the standard deviation of the yields of assets. In our case, all necessary calculations are made in Excel, which include: calculations of expected returns and calculations of the standard deviation of the yields of assets included in the portfolio. In this case, as the value of profitability, we will consider the forecast value for five years ahead.

The results are summarized in Table 1, sorting by increasing risk.

Table 1
Risks and returns of funds

№ of fund	Fund	σ	E(r)	Return/risk	Structure, %
1	Shares	0,1312	2,1	16,0061	32,4
2	Balanced	0,2127	2,994	14,0762	28,5
3	MICEX	0,2294	1,789	7,7986	15,7
4	Russian oil	0,3533	4,085	11,5624	23,4
Total					100

Table 1 shows that the least risky fund is the fund of shares, the most risky fund is the Russian Oil Fund. The MICEX Index Fund is relatively risky at relatively low profitability, but this does not mean that its presence in the portfolio becomes redundant, since we operate with data that are essentially probabilistic. Based on the yield / risk ratio for each fund, we form the portfolio structure, in proportion to the yield / risk ratio. The resulting portfolio consists of investments in funds in the following ratio: the Fund „MICEX Index“ should invest 15.7% of the total investment, the fund „Russian Oil“ should invest 23.4% of the total investment, a balanced fund should be invested 28.5% of the total amount of investments, 32,4% of the total amount of investments should be invested in the fund of shares.

The main principles for the formation of an optimal investment portfolio are:

- security of investments (stability of income, invulnerability in the market of investment capital);
- profitability of investments with a minimum level of risk;
- growth of investments;
- liquidity of investments.

Let's consider what portfolio yield will be received as a whole.

The expected return on the investment portfolio is defined as the weighted average of the expected returns on assets included in the portfolio:

$$\mathrm{E}(\, r_p \,) = \Sigma\, \mathrm{E}(r_i)\ Q_i\,, \qquad (10)$$

where: E (r_i) is the expected return of the i asset included in the portfolio, Q_i is the share of the value of the i asset in the total value of the portfolio (at market value at the time of portfolio compilation).

In our case: E(r_p)=2,1*0,324+2,994*0,285+1,789*0,157+4,085*0,234=2,7697, or 276,97%

This value is obtained for a time period of five years.

II. Annual return of the portfolio

By dividing by five, we get the expected value of the annual return of the portfolio:

276.97% / 5 = 55.39%, which is quite acceptable.

Accordingly, the monthly growth of the portfolio can be obtained as follows:

276.97% / 60 = 4.62%

Now we estimate the risk of the portfolio received.

The risk of the portfolio as a whole is measured by means of the variance and the standard deviation according to the formula:

$$\sigma_p = \sqrt{(\Sigma\Sigma Q_i Q_j cov_{ij}}), \quad (11)$$

where cov_{ij} – yield covariance for securities i and j; Q_i и $Q_j$and j are the shares of the i-th and j-th securities in the portfolio, respectively. The coefficient of variation is calculated by the formula:

$$cov_{ij} = p_{ij}\sigma_i\sigma_j , \quad (12)$$

Where, p_{ij} – coefficient of correlation between the yield of i and j of the unit.

Calculate the correlation coefficients using Excel, taking their values into Table 2.

Table 2

Coefficient of correlation

Coefficient of correlation	**Values**
ρ 12	0,9680
ρ 13	0,9924
ρ 14	0,9783
ρ 23	0,9705
ρ 24	0,9735
ρ 34	0,9673

Now calculate the coefficients of variation:

$cov_{11} = p_{11}\sigma_1\sigma_1$= 1,0*0,1312*0,1312=0,01721

$cov_{12} = p_{12}\ \sigma_1\sigma_2$= 0,9680*0,1312*0,2127=0,02701

$cov_{13} = p_{32}\sigma_1\sigma_3$= 0,9924*0,1312*0,2294=0,02987

cov_{14}= p_{14} $\sigma_1\sigma_4$= 0,9783*0,1312*0,3533=0,04535

cov_{22}= $p_{22}\sigma_2\sigma_2$= 1,0*0,2127*0,2127=0,04524

cov_{23}= $p_{23}\sigma_2\sigma_3$= 0,9705*0,2127*0,2294=0,04735

cov_{24}= $p_{24}\sigma_2\sigma_4$= 0,9735*0,2127*0,3533=0,07315

cov_{33}= $p_{33}\sigma_3\sigma_3$= 1,0*0,2294*0,2294=0,05262

cov_{34}= $p_{34}\sigma_3\sigma_4$ = 0,9673*0,2294*0,3533=0,07839

cov_{44}= $p_{44}\sigma_4\sigma_4$ = 1,0*0,3533*0,353=0,12482

Next, we determine the risk of a portfolio by the formula:

δp = (ΣΣ $Q_i Q_j cov_{ij}$) =
(32,4*32,4*0,01721+32,4*28,5*0,02701+…+23,4*23,4*0,12482)= 482,7038

Returning to the original dimension, we get:

482,7038/(100*100)=0,04827 or 4,83%.

Comparing the obtained value with the smallest (see Table 1), equal to 0.1312, we see: as a result of diversification, the portfolio risk became less in 0,1312 / 0,0483 = 2,7 times.

Let's make a general conclusion: The resulting portfolio has a yield of 55.39% per annum with a risk level of 4.83%.

Optimization of the investment portfolio in order to reduce its risk level at a given level of profitability is carried out on two parameters: covariance and diversification of its financial assets using elements of linear programming.

Then the linear programming problem will look like this.

Suppose we have a given level of profitability, equal to 55% per annum.

It is required to find such distribution of shares of the units entering into the portfolio, at which the lowest level of risk will be achieved.

In theory, one of the methods for solving such a problem, called the diversification of Markovts, is proposed.

The diversification of Markovets is based on the use of linear programming methods. In this case, the objective function and constraints are formed, and on their basis the built-in Lagrange function.

The objective function of this task.

σp_2=(ΣΣ $Q_i Q_j cov_{ij}$) → min (13)

Restrictions:

1) Average portfolio return

E_{cp} =Σ E (r_i) Q_i (14)

2) The normalization condition for the distribution

Σ Q_i=1 (15)

To solve this problem, it is necessary to form the Lagrange function:

$L = \Sigma\Sigma\ Q_i Q_j cov_{ij}\ +\ \lambda\ 1\ (\ \Sigma\ E(r_i)\ Q_i\ -\ E_{cp}) +\ \lambda\ 2\ (\Sigma\ Q_i\ -1),$ (16)

Where λ_i (i =1,2) – multiplier of Lagrange

A portfolio minimizing risk is determined by solving a system of equations:

$$\frac{\partial L}{\partial Q_i} = 0$$

$$\frac{\partial L}{\partial \lambda_i} = 0, \quad (17)$$

where: i = 1,2.

We rewrite the problem for our case if i, j = 1,2,3,4:

The objective function is equal to:

$$L = Q_{12}\ cov_{11} + Q_{22} cov_{22} + Q_{32} cov_{33} + Q_{42} cov_{44} + $$
$$+ 2\ Q_1 Q_2 cov_{12} + 2\ Q_1 Q_3 cov_{13} + 2\ Q_1 Q_4 cov_{14} + 2\ Q_2 Q_3 cov_{23} + 2\ Q_2 Q_4 cov_{24}$$
$$+ Q_3 Q_4 cov_{34} + \lambda_1\ (\ E\ (r_1)\ Q_1\ +\ E\ (r_2)\ Q_2\ +\ E\ (r_3)\ Q_3\ +\ E\ (r_4)\ Q_4 - E_{cp})$$
$$+ \lambda_2\ (\ Q_1 + Q_2 + Q_3 + Q_4 - 1) \quad (18)$$

Partial derivatives are equal to:

$$\frac{\partial L}{\partial Q_1} = 2\ Q_1 \sigma_{11} + 2\ Q_2 \sigma_{12} + 2\ Q_3 \sigma_{13} + 2\ Q_4 \sigma_{14} + \lambda_1\ E_1 + \lambda_2 = 0\ ;$$
$$\frac{\partial L}{\partial Q_2} = 2\ Q_1 \sigma_{12} + 2\ Q_2 \sigma_{22} + 2\ Q_3 \sigma_{23} + 2\ Q_4 \sigma_{24} + \lambda_1\ E_2 + \lambda_2 = 0\ ;$$
$$\frac{\partial L}{\partial Q_3} = 2\ Q_1 \sigma_{13} + 2\ Q_2 \sigma_{23} + 2\ Q_3 \sigma_{33} + 2\ Q_4 \sigma_{34} + \lambda_1\ E_3 + \lambda_2 = 0;$$
$$\frac{\partial L}{\partial Q_4} = 2\ Q_1 \sigma_{14} + 2\ Q_2 \sigma_{24} + 2\ Q_3 \sigma_{34} + 2\ Q_4 \sigma_{44} + \lambda_1\ E_4 + \lambda_2 = 0;$$
$$\frac{\partial L}{\partial \lambda_1} = Q_1 E_1 + Q_2 E_2 + Q_3 E_3 + Q_4 E_4 - E_{av} = 0;$$
$$\frac{\partial L}{\partial \lambda_2} = Q_1 + Q_2 + Q_3 + Q_4 = 0\ (19).$$

Presenting the system of equations in the matrix form:

$2cov_{11}$	$2cov_{12}$	$2cov_{13}$	$2cov_{14}$	E_1	1		Q_1		0
$2cov_{21}$	$2cov_{22}$	$2cov_{23}$	$2cov_{24}$	E_2	1		Q_2		0
$2cov_{31}$	$2cov_{32}$	$2cov_{33}$	$2cov_{34}$	E_2	1	*	Q_3	=	0
$2cov_{41}$	$2cov_{42}$	$2cov_{43}$	$2cov_{44}$	E_2	1		Q_4		0
E_2	E_2	E_2	E_2	0	0		λ_1		E_{cp}
1	1	1	1	0	0		λ_3		1

Fig. 7. Matrix representation of the Lagrange system (common form)

If we denote matrix by H and vector in the left by A and vector in the right by G, we gain a matrix equation:

H * A= G (20)

Below, on the figure 8, is the matrix representation of the equation (20) in the table form with the data from the equation system and the target function (18-19).

After solving the problem we obtain the required distribution vector:

A = H - 1 * G

0,03442	0,05402	0,05974	0,0907	2,1	1		Q_1		00
0,05402	0,09048	0,0947	0,1463	2,994	1		Q_2		00
0,05974	0,0947	0,10524	0,15678	1,789	1	*	Q_3		00
0,0907	0,1463	0,15678	0,24964	4,085	1		Q_4	=	00
2,1	2,994	0,789	4,085	0	0		λ_1		555
1	1	1	1	0	0		λ_3		11

Fig. 8. Matrix form of the Lagrange system (numerical form)

Multiplying the matrix by table, we get:

A^T ={17,2;20,6;45,6;16,6} – required distribution of the shares of divvies in the optimized portfolio.

Table 3. Comparison of structure of the basic and optimized portfolio

Stock number	**Stock**	**σ**	**$E(r_i)$**	**Return to risk**	**Basic portfolio share, %**	**Optimal share, %**
1	Shares	0,1312	2,1	16,0061	32,4	17,2
2	Balanced	0,2127	2,994	14,0762	28,5	20,6
3	MICEX Index	0,2294	1,789	7,7986	15,7	45,6
4	Rosneft	0,3533	4,085	11,5624	23,4	16,6
Total					100,0	100,0

***III.* Return of the optimized portfolio.**

Now we can calculate the return of the optimized portfolio:

$E(r_p)$ = 2,1*0,172+2,994*0,206+1,789*0,456+4,085*0,166=2,47186,или 247,2%.

This value is vital for the 5-year long period. Dividing by 5, we obtain the expected value of the annual return of the portfolio:

247,19% / 5 = 49,44%.

Hence, monthly portfolio growth is:

247,19% / 60 = 4,12%.

Now we calculate the optimized portfolio risk:

$\sigma_p = \sqrt{\Sigma\Sigma Q_i Q_j \text{cov}_{ij}}$ =(17,2*17,2*0,01721+17,2*20,6*0,02701+…+16,6*16,6*0,12482) =317,6554

Returning to the basic dimension we get:
317,6554/(100*100)=0,0317

In the percentage risk is equal to 3.17%.

Cncluding, one can say that portfolio can be optimized, but the return goes down by 55,39 - 49,44 = 5,95%, and the risk goes down by 4,83 -3,17=1,66%.

List of used literature:

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