MULTIFUNCTIONAL INVERSE SYNTHETIC APERTURE RADAR: MAIN PROCESSING ALGORITHMS

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***Abstract**: In this work ISAR (Inverse Synthetic Aperture Radar) multifunctional Inverse Synthetic Aperture Radar (ISAR) system and image reconstruction concept based radar parameters measurement is considered. Main geometric and kinematic characteristics are derived. LFM (Liner Frequency Modulated) emitted waveforms is used while modeling reflected ISAR returns Signal formation and image reconstruction are analytically described and interpreted as direct and inverse projection operation while image reconstruction is interpreted as totally motion compensation procedure. High precision image reconstruction algorithm based on spatial correlation is derived.*

***Key words**: ISAR, ISAR geometry, ISAR kinematics, Cross-correlation image reconstruction*

МНОГОФУНКЦИОНАЛНА РАДИОЛОКАЦИОННА СИСТЕМА СЪС СИНТЕЗИРАНА АПЕРТУРА: ОСНОВНИ ОБРАБОТВАЩИ АЛГОРИТМИ

***Анотация:** В тази доклад се представя концепция за възстановяване на изображения, получени в радиолокационна система с инверсна синтезирана апертура (ISAR), като се използват данни от измерванията на параметрите на обекта в системата. Линейно-честотно (LFM) модулиран сигнал се използва за моделиране на отразените от обекта сигнали. Изведени са основните геометрични и кинематични характеристики. Описани са аналитично процесите на формиране на ISAR сигнала и възстановяване на изображението, интерпретирани като права и инверсна проекционна операция. В допълнение, възстановяването на образа се интерпретира като пълна компенсация на фазите на сигнала, индуцирани от движението на обекта, т.е. пълна компенсация на движението на обекта. Предложен е високо-резолюционен алгоритъм за възстановяване на образа на базата на пространствена крос-корелация.*

1. Introduction

More than twenty years Inverse Synthetic Aperture Radar (ISAR) theory and practice, systems and technology are in focus of researches over the world. Methods and algorithms have been developed to meet ever growing requirements in respect of resolution and power budget. The implementation of ISAR concept will enlarge the aria of application and improve substantially the functionality of imaging radars and moving target recognition.

ISAR imaging is a technique to extract high quality images of moving objects from ISAR returns. The signal formation and image extraction processes are both coherent processes. The target is illuminated by a sequence of coherent pulses that are amplitude and phase modulated by the object surface an obtained by receiver during relative motion of the target with respect to the ISAR system, placed in the origin of the coordinate system of observation. High range resolution of the image can be achieved by ISAR transmitted signals with a large bandwidth. High azimuth resolution can be realized using large synthetic aperture length during relative motion of the object in respect of the ISAR system of observation.

A classical ISAR image reconstruction technique is a range-Doppler compression accompanying with phase compensation [1,2]. Based on the spectral description of ISAR signals high effective signal processing and image reconstruction methods as parametric and semi-parametric methods are suggested in [3,4]. Joint time-frequency transform for radar range-Doppler imaging and ISAR motion compensation via adaptive joint time-frequency technique and adaptive Fourier transform are presented in [5-7]. A range profiling technique for synthetic wideband radar is suggested in [8]. Image enhancement autofocusing algorithms for ISAR image processing based on entropy minimization and contrast maximization are developed in [9, 10].

Despite many detailed descriptions of imaging techniques in literature where the imaging process is decomposed into two main stages, motion compensation and image reconstraction it is not underlined that in its essence the image reconstruction is a motion or phase compensation, i.e. removing all signal phases induced by motion of the target. Moreover, in that sense the signal formation and image extraction can be considered as direct and inverse projections with one and the same operator. As a consequence the imaging algorithm can derived as a spatial correlation procedure that requires target parameters measurement, including current slant

range distance, radial velocity and position angles, elevation and azimuth angles. The present paper addresses these problems.
The main goal of the paper is to define signal polar formation and image extraction in the field of LFM ISAR data and based on the interpretation of signal formation and image extraction as direct and inverse space transformations.

The remainder of the paper is organized as follows. In Section II ISAR geometry and measuring target parameters are defined, in Section III a LFM waveform and ISAR signal formation is described. In Section IV an image reconstruction cross-correlation algorithm is derived. In Section V results of numerical experiments and their interpretation are given.

2. Geometry and target parameters measurement

The crucial step in development of ISAR imaging algorithms is implementation of precise target characteristic measurement as target position and the velocity. Target position is defined by range and two angular coordinates, relative to the radar position. The angular coordinates are elevation angle relative to the local horizontal plane, and azimuth angle, measured relative to North or to array broadside azimuth in case of phased arrays.

There are many time, frequency and phase methods to measure a radial distance to the target. The accuracy of slant range measurement is achieved by using wideband coherent emitted signals as LFM waveforms with an appropriate pulse repetition frequency that satisfies range unambiguity measurements requirements.

Radar angular measurements are made using mono-pulse receive antennas that produce simultaneous receive beams slightly offset in angle to either side of the transmit beam. Other angle-measurement techniques involve transmission and reception of multiple signals at different angles around the target with rotating search radar, Target angular position with scanning radar may be measured by finding the center of a series of pulse returns as the antennas weeps past the target.

Target radial velocity may be measured in two ways: from the Doppler-frequency shift of the received signal, and from multiple range measurements.
Measurements using Doppler-frequency shift almost always give significantly better accuracy than non-coherent processing of range measurements.

2.1. Two-dimensional ISAR geometry

Consider two dimensional ISAR scenario depicted in Fig.1. The current position of the mass-center, point B is defined by the vector $\mathbf{R}_{0'}(p)$, oriented on OB direction. The current position of arbitrary point D is defined by the vector $\mathbf{R}_{mn}(p)$, oriented on OD direction. The target is moving rectilinearly in a two-dimensional coordinate system *Oxy* of observation in the origin of which the ISAR system is placed.

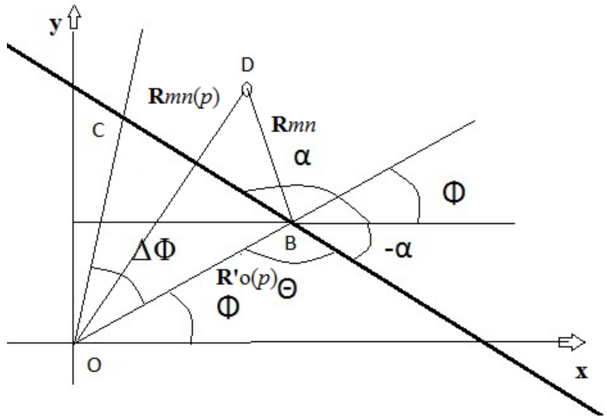

Fig. 1.

With the measurement unit of the ISAR system current trajectory parameters of the mass center of the target, point O', are evaluated as the slant range distance $R_{0'}(p)$ and azimuthal angle $\Phi(p)$ at the moment *p*, the index of emitted pulse. Projections $x_{0'}(p)$ and $y_{0'}(p)$ of $R_{0'}(p)$ on coordinates axes *Ox* and *Oy* are calculated by expressions

$$x_{0'}(p) = R_{0'}(p).\cos\Phi(p)$$
$$y_{0'}(p) = R_{0'}(p).\sin\Phi(p) \quad (1)$$

.
Based on trajectory equation of the target mass center

$$x_{0'}(p) = x_{0'}(0) + V_x.p.T_p;$$
$$y_{0'}(p) = y_{0'}(0) + V_y p.T_p, \quad (2)$$

where T_p is the period of measurement or pulse repetition period, $x_{0'}(0)$ and $y_{0'}(0)$ are initial coordinates of the mass center defined at the moment $p = 0$, coordinates of the linear vector velocity can be defined, i.e.

$$V_x = \frac{x_{0'}(p) - x_{0'}(0)}{p.T_p}$$
$$V_y = \frac{y_{0'}(p) - y_{0'}(0)}{p.T_p} \quad (3)$$

Then the module of the vector velocity is defined by

$$V = [V_x^2 + V_y^2]^{\frac{1}{2}}, \quad (4)$$

The angle of the vector velocity is defined by

$$\alpha = \arctan\frac{V_y}{V_x}. \quad (5)$$

2. LFM waveforms and ISAR signal model

LFM waveforms

The ISAR emits to the direction of a moving target a series of linear frequency modulated waveforms, each of which is described in a complex form as

$$\dot{S}(t) = \mathbf{rect}\frac{t}{T}\mathbf{exp}\left[-j(\omega t + bt^2)\right] \quad (6)$$

where $\omega = 2\pi\frac{c}{\lambda}$ is the angular carrier frequency; $c = 3.10^8$ m/s is the speed of the light; λ is the wavelength of the signal; T is the time duration of a LFM pulse; $b = \frac{2\pi\Delta F}{T}$ is the LFM rate. The bandwidth $(2\Delta F)$ of the transmitted pulse provides the dimension of the range resolution cell $\Delta R = c / 2\Delta F$.

LFM ISAR signal model

First, consider propagation of spherical electromagnetic waves and a generic point scatterer of the target with a position vector defined by

$$\mathbf{R}_{mn}(p) = \mathbf{R}_{0'}(p) + \mathbf{R}_{mn}. \quad (7)$$

where $\mathbf{R}_{0'}(p) = \left[x_{0'}(p), y_{0'}(p)\right]^T$ is the current vector position of the mass center defined in coordinate system *Oxy*, $\mathbf{R}_{ij} = \left[X_{mn}, Y_{mn}\right]^T$ is the distance vector of the generic point in the object's coordinate system $O'xy$.

Based on geometry in Fig. 1, equation (2) can be rewritten in matrix form as

$$\begin{bmatrix} x_{mn}(p) \\ y_{mn}(p) \end{bmatrix} = \begin{bmatrix} x_{0'}(p) \\ y_{0'}(p) \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} X_{mn} \\ Y_{mn} \end{bmatrix}, \quad (8)$$

where $x_{0'}(p) = x_{0'}(0) + V_x.p.T_p$, $y_{0'}(p) = y_{0'}(0) + V_y.p.T_p$, $V_x = V\cos\alpha$, $V_y = V\sin\alpha$, $X_{mn} = m.\Delta X$, $Y_{mn} = n.\Delta Y$.

From matrix equation (8) follows

$$\begin{aligned} x_{mn}(p) &= x_{0'}(p) + X_{mn}\cos\alpha + Y_{mn}.\sin\alpha \\ y_{mn}(p) &= y_{0'}(p) - X_{mn}.\sin\alpha + Y_{mn}.\cos\alpha \end{aligned}, \quad (9)$$

where α is the velocity angle.

Define a new coordinate system of wave propagation with a radial axis – line of sight of the mass center of the target, and orthogonal to the line of sight axis that lies on the wave front, and with the origin, point O' with initial coordinates $x_{0'}(0)$, $y_{0'}(0)$ in coordinate system *Oxy*. The angle between the line of sight and *Ox* axis is $\Phi(p)$ defined by $\Phi(p) = \arctan\frac{y_{0'}(p)}{x_{0'}(p)}$, with initial value $\Phi = \Phi(0) = \arctan\frac{y_{0'}(0)}{x_{0'}(0)}$, where $x_{0'}(0)$ and $y_{0'}(0)$ are initial coordinates of the mass center of the target. The axis $O'X$ of the target coordinate system is oriented along the vector velocity, thus the angle between $O'X$ and Ox is α.

Then the distance to the particular point scatterer is defined by

$$|\mathbf{R}_{mn}(p)| = \left[(x_{mn}(p))^2 + (y_{mn}(p))^2\right]^{\frac{1}{2}}. \quad (10)$$

Consider plane electromagnetic waves propagating in the particular radial direction. The scalars - angular frequency and LFM rate are transformed in co-linier vectors, angular vector velocity and angular vector acceleration with one and the same unit vector, pointed in the direction that the wave is traveling. Based on plane wave approximation (Fig.1) the module of the vector position of the generic point is defined by

$$|\mathbf{R}_{mn}(p)| = R_{0'}(p) + X_{mn}\cos[180 - \alpha + \Phi(p)] + Y_{mn}\sin[180 - \alpha + \Phi(p)] \quad (11)$$

or $|\mathbf{R}_{mn}(p)| = R_{0'}(p) - X_{mn}\cos[\alpha - \Phi(p)] + Y_{mn}\sin[\alpha - \Phi(p)]$.

The deterministic component of the ISAR signal reflected by the target’s generic point and registered in polar coordinates: radial range direction defined by unit vector **n** pointing in direction that the wave is traveling (fast time axis *t*), and azimuth polar coordinate angle on the cross range direction (discrete slaw time axis *p*) can be presented as

$$\dot{S}_{mn}(p,t) = a_{mn}.\mathbf{rect}\frac{t - t_{mn}}{T}\mathbf{exp}\left\{-j\left[\omega(t - t_{mn}) + b(t - t_{mn})^2\right]\right\} \quad (12)$$

where $\mathbf{rect}\frac{t - t_{mn}}{T} = \begin{cases} 1, & 0 \le \frac{t - t_{mn}}{T} < 1, \\ 0, & \frac{t - t_{mn}}{T} < 0, \text{and } \frac{t - t_{mn}}{T} \ge 1 \end{cases}$

where a_{mn} is the reflection coefficient of the *mn*-th point scatterer; $t_{mn} = \frac{2.|\mathbf{R}_{mn}(p)|}{c}$ is the time delay of the signal from the ***ij***-th point scatterer, $\mathbf{R}_{mn}(p) = \mathbf{R}_{0'}(p) + \mathbf{R}_{mn}$, where $\mathbf{R}_{0'}(p)$ is the current position vector to the mass center of the target, $\mathbf{R}_{mn}$ is the distance vector of the generic point. $t = [k_{mn\min}(p) + k]\Delta T$ is the time dwell the ISAR signal, fast time on the range direction, $k = \overline{1, [k_{mn\max}(p) - k_{mn\min}(p)] + K}$ is the sample number of a LFM pulse; $K = \frac{T}{\Delta T}$ is the full number of samples of the LFM pulse, ΔT is the time duration of a LFM sample, $k_{mn\min}(p) = \left\lceil \frac{t_{mn\min}(p)}{\Delta T} \right\rceil$ is the number of the radar range bin where the signal, reflected by the nearest point scatterer of the target is detected, $t_{mn\min}(p) = \frac{2.|\mathbf{R}_{mn\min}(p)|}{c}$ is the minimal time delay of the ISAR signal reflected from the nearest point scatterer of the target, $K(p) = k_{mn\max}(p) - k_{mn\min}(p)$ is the relative time dimension of the target; $k_{mn\max}(p) = \left\lceil \frac{t_{mn\max}(p)}{\Delta T} \right\rceil$ is the number of the radar range bin where the signal, reflected by farthest point scatterer of the target is detected; $t_{mn\max}(p) = \frac{2|\mathbf{R}_{mn\max}(p)|}{c}$ is the maximum time delay of the ISAR signal reflected from the farthest point scatterer of the target; $R_{mn}(p)$ is the module of the range distance vector to the *mn*-th point scatterer of the target.

The deterministic components of the ISAR signal return from the target are defined as a superposition of signals reflected by all point scatterers, i.e.

$$\dot{S}(p,t) = \sum_{m,n} \dot{S}_{mn}(p,t) = \sum_{m,n} a_{mn} \mathbf{rect} \frac{t - t_{mn}(p)}{T} \mathbf{exp}\left\{- j\left[\omega\left(t - t_{mn}(p)\right) + b\left(t - t_{mn}(p)\right)^2\right]\right\}. \quad (13)$$

Frequency demodulation (de-chirping) of the ISAR signal return is performed by multiplication of the left and right parts with a complex conjugated emitted waveform, i.e.

$$S(p,t).\exp[-j(\omega t + bt^2)] = \hat{S}(p,t) = \sum_{m,n} a_{mn}.\exp\left\{j[\omega(t - t_{mn}(p)) + b.(t - t_{mn}(p))^2]\right\}\exp[-j(\omega t + bt^2)] \quad (14)$$

which yields

$$\hat{S}(p,t) = \sum_{m,n} a_{mn}.\exp\left\{- j[(\omega + 2bt)t_{mn}(p) - b(t_{mn}(p))^2]\right\} \quad . \quad (15)$$

Denote $\omega(t) = \omega + 2bt$, where ω is the carrier angular frequency, $t = k.\Delta T$ is the discrete time parameter, where $k = \overline{0, K-1}$ can be considered as the frequency sample number as well, ΔT is the sample time duration. Then the current discrete angular frequency can be expressed as $\omega(k) = \omega + 2b.k.\Delta T$.

Then expression (15) can be rewritten as

$$\hat{S}(p,t) = \sum_{m,n} a_{mn}.\exp\left[- j\left(2\frac{\omega(t).|\mathbf{R}_{mn}(p)|}{c} - b\left(\frac{2.|\mathbf{R}_{mn}(p)|}{c}\right)^2\right)\right], \quad (16)$$

In discrete form (16) can be written as

$$\hat{S}(p,k) = \sum_{m,n} a_{mn}.\exp\left[- j\left(2\omega(k).\frac{|\mathbf{R}_{mn}(p)|}{c} - b.\left(\frac{2.|\mathbf{R}_{mn}(p)|}{c}\right)^2\right)\right], \quad (17)$$

where

$|\mathbf{R}_{mn}(p)| = R_{0'}(p) - X_{mn}\cos[\alpha - \Phi(p)] + Y_{mn}\sin[\alpha - \Phi(p)]$, $X_{mn} = m.\Delta X$, $Y_{mn} = n.\Delta Y$.

$R_{0'}(p) = \sqrt{[x_{0'}(p)]^2 + [y_{0'}(p)]^2}$, $\Phi(p) = \arctan\frac{y_{0'}(p)}{x_{0'}(p)}$, $x_{0'}(p) = x_{0'}(0) + V_x.p.T_p$ $y_{0'}(p) = y_{0'}(0) + V_y p.T_p$

.

Expression (17) can be considered as a direct projection of a two-dimensional (2D) image function on to 2D ISAR signal plane with projection operator, $\exp\left[- j\left(2\omega(k).\frac{|\mathbf{R}_{mn}(p)|}{c} - b.\left(\frac{2.|\mathbf{R}_{mn}(p)|}{c}\right)^2\right)\right]$.

ISAR image reconstruction as cross-correlation procedure

ISAR image reconstruction is an inverse projecting procedure. Based on the expression (17), the target image can be extracted by the following equation

$$a_{mn} = \sum_{p,k} \hat{S}(p,k).\exp\left[j\left(2\omega(k).\frac{|\mathbf{R}_{mn}(p)|}{c} - b.\left(\frac{2.|\mathbf{R}_{mn}(p)|}{c}\right)^2\right)\right], \quad (18)$$

which can be rewritten as

$$a_{mn} = \sum_{p,k} \hat{S}(p,k).\exp\left[j\left(-b.\left(\frac{2.|\mathbf{R}_{mn}(p)|}{c} \right)^2 \right) \right].\exp\left[j\left(2\omega(k).\frac{|\mathbf{R}_{mn}(p)|}{c} \right) \right], \qquad (19)$$

where $m = 0 - (M - 1)$, $n = 0 - (N - 1)$, M is the full number of point scatterers on OX axis, N is the full number of point sctterers on OY axis in the object space.
Expression (18) can be considered as an inverse projection of a 2D ISAR signal into 2D image function by projection operator, $\exp\left[j\left(2\omega(k).\frac{|\mathbf{R}_{mn}(p)|}{c} - b.\left(\frac{2.|\mathbf{R}_{mn}(p)|}{c} \right)^2 \right) \right]$.

Moreover, multiplication $\hat{\hat{S}}(p,k) = \hat{S}(p,k).\exp\left[j\left(-b.\left(\frac{2.|\mathbf{R}_{mn}(p)|}{c} \right)^2 \right) \right]$ removes quadratic phases from demodulated ISAR signal registered in each range cell, $k = 0 - (N - 1)$, in the interval $p = 1 - M$.

The image function can be rewritten as

$$\begin{aligned} a_{mn} &= \sum_{p,k} \hat{\hat{S}}(p,k).\exp\left(j\frac{2\omega(k)}{c}|\mathbf{R}_0(p)| \right) \\ &\times \exp\left[j\left(-\frac{2\omega(k)}{c}.\cos(\alpha - \Phi(p)).X_{mn} + \frac{2\omega(k)}{c}.\sin(\alpha - \Phi(p)).Y_{mn} \right) \right] \end{aligned} \qquad (20)$$

In addition, multiplication with first exponential term, $\exp\left(j\frac{2\omega(k)}{c}|\mathbf{R}_0(p)| \right)$ in (20), compensates the radial displacement of the mass-center of the object induced by radial velocity.
Multiplication with exponential term

$$\exp\left[j\left(-\frac{2\omega(k)}{c}.\cos(\alpha - \Phi(p)).X_{mn} + \frac{2\omega(k)}{c}.\sin(\alpha - \Phi(p)).Y_{mn} \right) \right],$$

compensates radial displacements of *mn*th point scatterer from the object space. Consequently, the computational procedure for image extraction from ISAR returns can be considered as total motion compensation of all phase components in ISAR signals induced by all kind of object motion.

Conclusion

In the present work ISAR multifunctional Inverse Synthetic Aperture Radar (ISAR) system and image reconstruction concept based radar parameters measurement have been analysed. Main geometric and kinematic characteristics are derived. Signal formation and image reconstruction are analytically described and interpreted as direct and inverse projection operation while image reconstruction is interpreted as totally motion compensation procedure. LFM emitted waveforms have been used to describe the structure of reflected ISAR returns. High precision image reconstruction algorithm based on spatial cross-correlation has been derived.

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