ГРЪЦКИТЕ БУКВИ И УПРАВЛЕНИЕ НА РИСКА НА ДЕРИВАТИВНИ ИНСТРУМЕНТИ

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GREEKS AND DERIVATIVE PRODUCTS' RISK MANAGEMENT

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Анотация: *Настоящата статия е фокусирана върху изследването на гръцките букви и тяхното използване в управление на риска на финансовите деривативи (по-специално пазарната чувствителност на опциите). Накратко е обоснована важността на управлението на риска и финансовите деривативни инструменти и влиянието, което те оказват на институционално ниво (банки и финансови институции) и на корпоративно ниво (бизнес инвестиране). Изследването и управлението на тези финансови инструменти се осъществява изключително трудно. В статията са представени някои подходи за оценка и управление. С цел да се демонстрира практическото приложение на изследваните методи и средства, теоретичните обосновки и описания са допълнени и разширени с продходящи примери.*

Ключови думи: *Гръцки букви, Деривативни инструменти, Управление на риска, Волатилност, Биномиален модел, Модел на Блек-Скоулс-Мертън, Симулации на Монте Карло, Опционни стратегии, Long Call, Short Call, Bull Spread, Bear Spread, Long Strangle, Short strangle, Long Butterfly, Short Butterfly.*

Annotation*: This article is focused on a detailed study of the Greeks and their use in risk management of derivatives (mainly options' sensitivity). The importance of derivatives and risk management in the field of finance is briefly explained. These products influence significantly both on institutional (banks and financial organizations) and corporate (business investment) level. However, these products turn out to be one of the most difficult to assess and manage and in the article are presented some methods, which simplify these processes. All theoretical explanations rely on numerical examples in order to demonstrate the practical application of the methods and tools studied.*

Key words*: The Key Greeks, Derivatives, Risk Management, Volatility, Binomial Model, Black-Scholes-Merton Model, Monte Carlo Simulations, Options Strategies, Long Call, Short Call, Bull Spread, Bear Spread, Long Strangle, Short strangle, Long Butterfly, Short Butterfly.*

Introduction

The aim of this work is to study and explain the importance of derivatives and risk management in the field of finance. In this paper we will focus on the management of the market risk since precisely this is the type of risk that you can control with the help of the Greeks [Banks&Siegal'2006; Haug'2007].

This work is based on the options as one of the most common derivative products. Their main difference with other derivatives is that they offer the option buyer the right to exercise or not to exercise the option and only suppose an obligation for the option seller. To assess and manage options one has to understand how to determine their value and risk and, therefore, how to manage said risk.

The first thing to be explained in this article is what are the Greeks and how they relate to the risk, furthermore the paper will try to give a detailed explanation to the most used option pricing models and ultimately it will explain how you can manage the risk of an option knowing its Greeks and using its pricing model.

1. The Key Greeks

The Greeks in financial mathematics are indicators of individual risk factors (such as pass of time, volatility, underlying asset's price, etc.) and serve to measure the sensitivity of a product towards changes in one of these factors. We use the "Greeks" designation because each of these factors is indicated by a letter of the Greek alphabet. Importantly, the Greeks are partial derivatives, which means that they measure how does our position change if only one of the factors deviates while the others remain constant (ceteris paribus).

The main Greeks we use to measure risk are delta, gamma, vega, theta and rho.

- Delta (△)

Delta can be calculated as it is shown: $\frac{\partial \Pi}{\partial S}$.

Its interpretation is as follows: when the option price increases by a certain amount, the portfolio value or price of the instrument, changes in △% of this amount.

The delta is also often interpreted as a hedge against the underlying spot.

- Gamma (Γ)

Gamma is calculated like this: $\frac{\partial \Delta}{\partial S} = \partial^2\Pi/\partial S^2$

It indicates the speed of changes in the premium when there are changes in the underlying asset's price. Gamma also provides information on the volatility of the underlying asset.

- Vega (V)

To calculate Vega we use: $\frac{\partial \Pi}{\partial \sigma}$

It shows the sensitivity of the option price to changes in the volatility. If you look at the Vega of a portfolio of derivatives, it would indicate the rate of changes of the portfolio's value with respect to the volatility of the underlying asset.

The basic interpretation of Vega is that if it is high in absolute value, the value of the portfolio is very sensitive to small changes in the volatility and if it is low in absolute value, the changes in the volatility have a relatively low impact on the value of the portfolio.

- Rho (ρ)

Rho is calculated as: $\frac{\partial \Pi}{\partial r}$

This Greek indicates the sensitivity of the price of an option to changes in the interest rate. In other words, it helps us determine the risk in monetary terms.

As for the interpretation of the value of Rho one must know that it increases in absolute value when the option is at the money and decreases when the option is out of the money.

- Theta (θ)

Theta is calculated using the following mathematical expression: $\frac{\partial \Pi}{\partial x}$

Theta is an indicator of changes in the option price with respect to time. Theta could also refer to the time decay of the portfolio. It measures the aging of an option.

Theta is usually negative for an option. This is because as time to maturity decreases and everything else remains constant, the value of the option tends to decrease.

2. Types of Options

As we have mentioned it is important to know that not all options have the same features and therefore they are not affected by market changes in the same way. Mainly we could distinguish between standard options (plain vanilla) and non-standard options (exotic).

1. Standard Options (plain vanilla) are the most common in organized markets. They can be divided into two categories: European and American options. They differ in that the first ones must be exercised only on the specific date stated in the contract, however the American ones can be exercised at any time until maturity [Wolfiner'2008, 2013].

2. Exotic options were created by the need to meet the requirements of the individual investors. These options are not usually listed on organized markets so they are less liquid than traditional options. Among the exotic options we can distinguish many types of options [Rubinstein&Reiner'1992, Weert'2008, Wolfinger'2013]. Some of the most important divisions within the exotic option are:

- Path dependent – the value of this type of derivatives does not depend only on the value of the underlying asset on the expiry date but also on the path taken by the risky asset from the time of recruitment to maturity. This group can be subdivided in Asian options, Extremum dependent, Stair Ways, Shorts and Chooser.

Within this group the best known are the Asian options. They are options in which the underlying asset's value is determined by an arithmetic mean of the values obtained by the risky asset over the life of the option and the strike is calculated using the average arithmetic underlying asset's values achieved over the life of the option.

- Compound options - these are options on options. In these contracts there are two strike prices, two underlying securities and two maturities. The different possible combinations are: call on call, call on put, put on call and put on put.
- Options with singular payment – the quantities agreed in the contract will only be received if in the end the position of the asset given the strike is in accord to the conditions of the contract. Sometimes even the premium is paid at the end only if the option is exercised.

- Rainbow options - these are options which have more than one risky (underlying) assets.

Due to the diversity of options in practice we must apply various pricing models but among them there are three that can be applied when pricing most of the options and also they are giving far more reliable results than the other ones. These three models are the binomial model [Georgiadis'2011], the Black-Scholes-Merton [Bodie et al.'2008, Hull'2008, Hong-Yi et al.'2010] and the Monte Carlo simulations [Kroese et al.'2011].

3. Main Options Pricing Models and Estimation of the Greeks

3.1. The Binomial Model

This model is static in discontinuous time. One of its main assumptions is risk neutrality, in other words the relative risk of the assets is not taken into account. Another assumption to consider is the lognormal behavior of the stock returns.

Keep in mind that the binomial model is used primarily to assess exotic options and American options.

To give a more detailed and simple tutorial we will study each part of the model following an example: Let's imagine there is an option offered in the market which has the following characteristics:

Asian call, strike=90, the risk free rate equals zero (in order to simplify calculations) and the expiration date is in 2Δt. The option is negotiated on an underlying asset S0=100 which is a financial asset that does not offer any dividends. The coefficients of variation are 0.5 and we know that u=d=2.

The first step in pricing an option is to build the tree that the underlying asset is following. To branch the values taken by the risky asset at all times we must first define the coefficients of variation: u (indicates a rise in the price) and d (indicates a low in the price) [Kolb'2002]. Please see Figure 1.

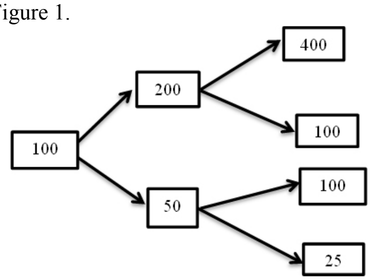

Figure 1. To obtain each of these values we have been multiplying S0 per u and per d. This is how we have obtained the results for the following positions: u=200, d=50, uu=400, ud=100, du=100 and dd=25.

The second step is to calculate the implicit probabilities. These probabilities we calculate as follows:

Pu= Pd= 1 – Pu

In our particular case we have: Pu= and Pd=

The last step consists in calculating the actual price of the option using the previous probabilities and the non-recombinant tree of the risky asset. Please see Figure 2.

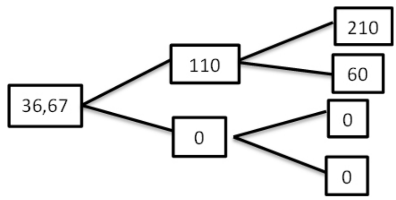

Figure 2

To calculate the values in 2Δt we have done the following operations:

$\left(\frac{200+400}{2}-90\right)^{+} * e^{-r\Delta t} = 210$

$\left(\frac{200+100}{2}-90\right)^{+} * e^{-r\Delta t} = 60$

$\left(\frac{50+100}{2}-90\right)^{+} * e^{-r\Delta t} = 0$

$\left(\frac{25+50}{2}-90\right)^{+} * e^{-r\Delta t} = 0$

In our case $e^{-r\Delta t} = 1$ because the interest rate equals zero.

The superscript sign + next to the parentheses indicates that we will only take into account the positive values.

The rest of the numerical results (at Δt and at t= 0) have been obtained like this:

$110 = (210*Pu + 60*Pd)^{+} * e^{-r\Delta t}$

$36,67 = (110*Pu + 0*Pd)^{+} * e^{-r\Delta t}$

This model provides a fairly simple way to calculate the delta but calculating the rest of the main Greeks with the Binomial Model would be difficult. However, we can now show how the delta hedging portfolio in t=0 should be constructed. To create this portfolio we use the underlying asset's binomial tree and the Asian call's binomial tree to create a system of equations as follows:

$$\begin{cases} X + 200\triangle = 110 \\ X + 50\triangle = 0 \end{cases}$$

Solving the system we obtain:

△= 11/15 or which is the same 0,73333 units of the underlying asset. We are talking about a call and △ is positive, so it indicates us the quantity of underlying assets we need to sell.

X= -110/3 represents the quantity that we must invest in the risk free asset.

Now we have the hedging portfolio

Sell 11/15 units of the underlying asset= 11/15 * 100€ = 220/3€
Invest 110/3€ in the risk free asset = -110/3€ } 36,67€

Buying a call means that we must pay so we indicate this pay off as € -36.67 and the hedging portfolio is worth a total of €36.67, together they sum zero, so we have made the perfect delta hedging. Importantly, in each △t we have to recalculate the hedging portfolio and change it so that we continue having the risk of change in valuation removed.

3.2. The Black-Scholes-Merton Model

This is a mathematical model used to value options. It is a particular case of the binomial model, transformed into continuous time.The most important hypothesis of the model are: perfect capital markets, constant free risk rate, no restrictions on short selling, the shares pay no dividends, the actions follow a complex stochastic process, called Ito process. The main assumption that we do is also one of the greatest weaknesses of the model. We assume that the volatility is a constant. We should protect ourselves from this assumption.

This valuation system is intended primarily for European options. To price options we need the following formulas:

$$C_0 = S_0 * \varphi(d1) - k e^{-rt} * \varphi(d2)$$

$$P_0 = \varphi(d1) * F e^{-rt} - \varphi(d2) * k e^{-rt} + k e^{-rt} - F * e^{-rt}$$

$$d1 = \frac{\ln\left(\frac{S0}{k}\right) + \left(r + \frac{\sigma^2}{2}\right) * T}{\sigma * \sqrt{T}} \qquad d2 = d1 - \sigma\sqrt{T}$$

$$\theta = rH - \tfrac{1}{2} * \sigma^2 * S_0^2 * \Gamma$$

To calculate the rest of the Greeks we use the Table1.

Asset\Greeks	△	Γ	V
Free risk asset	0	0	0
Underlying asset	1	0	0
Call	$\varphi(d1)$	$\frac{\phi^1(d1)}{S0 * \sigma * \sqrt{T}}$	$S_0 * \varphi^1(d1) * \sqrt{T}$
Call on futures	$\varphi(d1)*$	$\frac{\phi^1(d1)}{S0 * \sigma * \sqrt{T}} * e^{-rt}$	$S_0 * \varphi^1(d1) * \sqrt{T} * e^{-rt}$
Put	$-(1-(\varphi(d1))$	$\frac{\phi^1(d1)}{S0 * \sigma * \sqrt{T}}$	$S_0 * \varphi^1(d1) * \sqrt{T}$
Put on futures	$-(1-(\varphi^{(d1)*}))$	$\frac{\phi^1(d1)}{S0 * \sigma * \sqrt{T}} * e^{-rt}$	$S_0 * \varphi^1(d1) * \sqrt{T} * e^{-rt}$

Table 1

We will now give an example of how to price a European option and how to calculate its Greeks using the Black-Scholes-Merton.

Let's say there is a put option on a future with F=1000 €, the σ=40 % (volatility) and the interest rate is zero (to simplify calculations), time to maturity is ¼ years. With some help from the table of the Normal distribution we can find the following values:

$\Phi(d1)= 0{,}5398 \quad \varphi(d2)= 0{,}4602$

$P_{0=} 0{,}5398*1000 - 0{,}4602*1000 + 1000 - 1000= 79{,}6€$

$\triangle put= -(1-\varphi(d1))= -0{,}4602$

$\varphi^1(d1)= (1/\sqrt{2}*\pi)*e^{\frac{-d1^2}{2}} = 0{,}3969525475$

$\Gamma= \frac{\phi^1(d1)}{F\sigma\sqrt{T}} * e^{-rt} = 0{,}001984763$

$V= F*\varphi^1(d1)*\sqrt{T} *e^{-rt} = 198{,}4762738$

$\theta= rH- ½ *\sigma^2*S_0^2*\Gamma= -158{,}78104$

Let's see what happens if in this case we only apply a delta hedging strategy but we don't consider the other Greeks and in a week the future is to be worth € 1,050 and volatility changes to 30%.

As the option's underlying asset is a future the delta hedge portfolio is static and very easy to calculate:

Buy 0,4602 futures= 0€
Borrow 76,9€= +76,9€ } 76,9€

The payoff of a bought put option is equivalent to € -76.9 and the value of the delta hedge portfolio is + € 76.9, they sum zero, therefore, we have neutralized the delta.

Now we will see how did the value of our portfolio "H", compound by the put option and the hedging portfolio, change given the new conditions. In order to avoid confusion with the symbol indicating the variation of the amounts and delta we will write the Greek this time as δ. Note that since we've hedged delta now $\delta*(\triangle F)= 0$

$\triangle H= \theta*(\triangle t) + \delta*(\triangle F) + V*(\triangle\sigma) + ½* \Gamma*(F)^2$

Solving the equation we obtain △H= -20,42015517€, which means that by not taking into account the other Greeks while constructing the hedge portfolio we will have lost 20,4 €.

3.3. The Monte Carlo Method

It is a non-deterministic method used to evaluate investment projects or products derived by picking a random sample. It is based mainly on statistical studies using technological advances to meet the own random behavior of the systems we want to analyze.

The simulation involves the development of a very large number of trajectories. We choose a very large sample so that the result shall be closer to reality. With computer assistance we obtain specific values of the variables studied. Repeating this process *n* times, we achieve *n* observations which help us to understand the functioning of the system. The greater the number of experiments is the more accurate said system becomes.

4. Hedging or Speculation?

The perfect hedging allows us to eliminate all the risk but it also takes away our chance to gain profits. On the other hand, the speculation occurs when an investor wants to obtain benefits derived from their personal assessment of market developments, creating a risk exposure profile. Such operations can be carried out both in the cash market and the derivatives.

The derivatives market, however, has two great advantages for the realization of speculation: it focuses on the price differences in different time periods and it allows the performing of complex strategies, impossible to perform in the cash market. In addition, options allow us to speculate without having to bet on a particular value, just make one bet downward or upward to win [Sanz Caballero'2000]. Here we will discuss some of the best known strategies.

Expectations according to which the investors decide whether to perform either strategy can be divided into two groups: bullish (imply that the investor expects that the variable in question increases) and bearish (the investor expects the variable in question to decrease).

Here we present (Please see below the Table 2) a table that helps us compare all strategies according to their basic characteristics:

Strategy	Expectative on the price	Expectative on the volatility	Potential profit	Potential loss	Effect of the time
Long call	Bullish	Bullish	Ilimited	Limited	Negative
Short call	Bearish	Bearish	Limited	Ilimited	Positive
Bull spread	Bullish	Neutral	Limited	Limited	Mixed effect
Bear spread	Bearish	Neutral	Limited	Limited	Mixed effect
Long strangle	Bullish	Bullish	Ilimited	Limited	Negative
Short strangle	Neutral	Bearish	Limited	Ilimited	Positive
Long butterfly	Neutral	Bearish	Limited	Limited	Positive
Short butterfly	Neutral	Bullish	Ilimited	Limited	Negative

Table 2

As mentioned these strategies help us gain profits based on certain predictions but they also allow us to limit our risk. Overall, speculative strategies can be exploited both to earn and to protect us from danger.

Conclusions

Risk management is crucial when making financial investments. It is useful for both the individual investor and the institutional investors. Often excessive investment risk is produced due to operations with derivatives that are complicated to handle.

The main reasons for the investors to fail when performing a certain market strategy (i.e. with options) is that they use derivatives without understanding well their main functions.

In order to lower their risk and make their market predictions more reliable investors need to study the derivatives' Greeks.The calculation of these indicators nowadays is not complicated as pricing models such as Black-Scholes – Merton and the Binomial model present us fast and effective ways to calculate the main Greeks with sufficient reliability.

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