WAVEFORMS, WAVES AND MATHEMATICAL MODELING OF RADAR SIGNAL FORMATION PROCESS

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ВЪЛНОВИ ФОРМИ, ВЪЛНИ И МАТЕМАТИЧЕСКО МОДЕЛИРАНЕ НА ПРОЦЕСА НА ФОРМИРАНЕ НА РАДАРНИ СИГНАЛИ

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Abstract: *In the present work theoretical description of waveforms, waves and mathematical model of radar signal formation is addressed. Based on the complex exponential function continuous and finite waveforms, waves and radar signals are analytically described and graphically illustrated. Radar imagining geometry is presented and analytical described.*

Key words: *radar waveforms, electromagnetic wave, radar signal modeling*

1. Introduction

Recently imaging radar technology attracts exceptional attention. Imaging radar is very important for many military and civilian applications including automatic target recognition (ATR) of non cooperative target [1-5], battlefields awareness [6], development as well as maintenance of low observable aircrafts and target characterization, planet imaging in radio astronomy [7, 12]. Compared with conventional radars, imaging radars improve detection and tracking performance, and exclusive target identification.
Imaging radars utilize range-Doppler principle to obtain and desired image of the object. The range resolution of the radar image is realized by the bandwidth of the transmitted radar signal, whereas the cross-range resolution is achieved from the gradient of the Doppler frequency spectrum generated by the relative displacement of the target with respect to the radar system of observation. It is a vast reason to deep the knowledge in principles of electromagnetic waveforms generation, wave propagation and radar signal formation. The main objectives of the paper:

1. Mathematical Models of Electromagnetic Continuous and Finite Waveforms and Microwaves.
2. Radar Signal Formation
3. Radar Imaging Geometry

The paper is organized as follows. In Section 2 mathematical models of electromagnetic continuous and finite waveforms and microwaves. In Section 3 radar signal formation is considered. In Section 4 radar imaging geometry is described. In Section 4 conclusions are made.

2. Waveforms and wave processes

a. Continuous waveform and waves

Continuous waveform

The waveforms are electrical oscillations called signals and generated by generators. The waveforms generate electromagnetic waves while propagating through material and immaterial environment. Both of them can be monochromatic (unmodulated) and wide bandwidth (modulated) waveforms. The waveforms can be divided into continuous waveforms and finite waveforms. While propagating continuous waveforms cause continuous waves whereas finite waveforms cause finite waves.

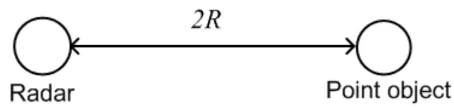

Fig.1.

Consider radar generated waveforms and emitted electromagnetic waves to a point object placed at distance R, reflected electromagnetic waves to the radar (Fig.1). A mathematical model of a continuous waveform with identity amplitude is the complex exponential function

(1) $$\dot{s}(t) = \exp(j\omega t) = \cos \omega t + j \sin \omega t$$

where $\mathrm{Re}[\exp(j\omega t)] = \cos \omega t; \mathrm{Im}[\exp(j\omega t)] = \sin \omega t$;

$\omega = \frac{2\pi c}{\lambda}$ is the angular frequency; $c = 3.10^8$ m/s denotes the speed of light; λ denotes the wavelength. The imaginary part of the waveform is presented in Fig. 2.

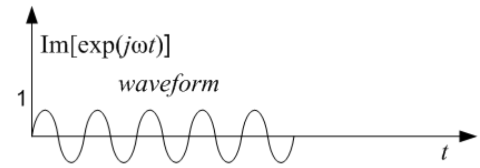

Fig. 2

Continuous wave

A mathematical model of a continuous wave with identity amplitude is the complex exponential function

(2) $$\tilde{s}(t) = \exp(j\omega(t - t_R)) = \cos \omega(t - t_R) + j \sin \omega(t - t_R)$$

where $\mathrm{Re}[\exp(j\omega(t - t_R))] = \cos(\omega(t - t_R)); \mathrm{Im}[\exp(j\omega(t - t_R))] = \sin(\omega(t - t_R))$,

$t_R = \frac{R}{c}$ is the time delay of the wave front measured on the range direction *R*.

Substitute $\omega = \frac{2\pi c}{\lambda}$ in (2), then

(3) $$s(t) = \exp\big(j(\omega t - kR)\big) = \cos(\omega t - kR) + j\sin(\omega t - kR),$$

where $k = \frac{2\pi}{\lambda}$ is the wave number.

In Fig. 3 an imaginary part of the continuous wave process as a function of the time measured in a particular range distance *R* is presented.

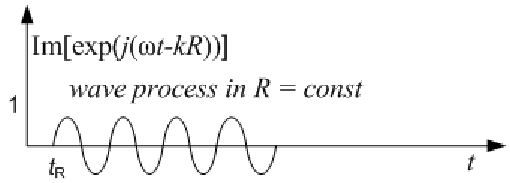

Fig. 3

The distribution of the wave on the range direction *R* in particular moments t_1, t_2 and t_2 is presented in Fig. 4. The displacement of the wave front in moments t_1, t_2 and t_2 can be observed.

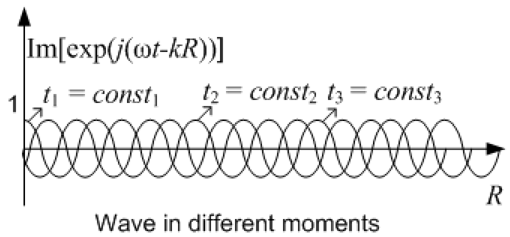

Fig. 4.

b. Finite waveforms and waves

- Finite waveforms

Finite monochromatic waveform

Consider finite monochromatic waveform. The mathematical model of a single finite monochromatic waveform with unity amplitude is the complex exponential function

(4) $$\dot{s}(t) = \mathbf{rect}\frac{t}{T}\mathbf{exp}(j\omega t)$$

where

$$(5)\qquad \mathbf{rect}\frac{t}{T} = \begin{cases} 1, 0 \le \frac{t}{T} < 1, \\ 0, \frac{t}{T} < 0 \\ 0, \frac{t}{T} \ge 1 \end{cases}$$

where T is the time duration of the finite monochromatic waveform.

Finite linier frequency modulated (LFM) waveform

Consider finite linier frequency modulated (LFM) waveform. The mathematical model of a single finite LFM waveform with unity amplitude is the complex exponential function

$$(6)\qquad \dot{s}(t) = \mathbf{rect}\frac{t}{T}\mathbf{exp}\left[j(\omega t + bt^2)\right]$$

where T is the time duration of the LFM waveform, b is the LFM rate.

$$(7)\qquad \mathbf{rect}\frac{t}{T} = \begin{cases} 1, 0 \le \frac{t}{T} < 1, \\ 0, \frac{t}{T} < 0 \\ 0, \frac{t}{T} \ge 1 \end{cases} \quad \text{is the rectangular function.}$$

Denote

$$(8)\qquad \mathrm{Re}\left[\dot{s}(t)\right] = \mathbf{rect}\frac{t}{T}\mathbf{cos}\left[j(\omega t + bt^2)\right], \text{ and } \mathrm{Im}\left[\dot{s}(t)\right] = \mathbf{rect}\frac{t}{T}\mathbf{sin}\left[j(\omega t + bt^2)\right].$$

c. Waveform train

Monochromatic waveform train

The mathematical model of a monochromatic waveform train with unity amplitude is the complex exponential function

$$(9)\qquad \dot{s}(t) = \sum_{t}\mathbf{rect}\frac{t}{T}\mathbf{exp}(j\omega t)$$

where $\tilde{t} = t \bmod T_p$ is the slow time, $t = \tilde{t} - pT_p$ is the fast time, T_p is the pulse repetition period, p is the number of the generated waveform (pulse).

LFM waveform train

The mathematical model of a finite LFM waveform train with unity amplitude is the complex exponential function

$$(10)\qquad \dot{s}(t) = \sum_{t}\mathbf{rect}\frac{t}{T}\mathbf{exp}\left[-j(\omega t + bt^2)\right]$$

where $\tilde{t} = t \bmod T_p$ is the slow time, $t = \tilde{t} - pT_p$ is the fast time, T_p is the pulse repetition period, p is the number of the generated waveform (pulse).
Denote

(11) $\quad \mathrm{Re}[\dot{s}(t)] = \sum_t \mathbf{rect}\frac{t}{T}\mathbf{cos}\left[j(\omega t + bt^2)\right]$, and $\mathrm{Im}[\dot{s}(t)] = \sum_t \mathbf{rect}\frac{t}{T}\mathbf{sin}\left[j(\omega t + bt^2)\right]$

In Fig. 5 the imaginary part of the finite LFM waveform train with unity amplitude is presented.

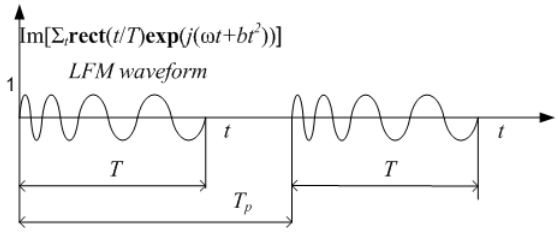

Fig. 5

d. Finite waves

Monochromatic wave
A mathematical model of a single finite monochromatic wave with unity amplitude is the complex exponential function

(12) $$\dot{s}(t,R) = \mathbf{rect}\frac{t-t_R}{T}\mathbf{exp}\left[j\omega(t-t_R)\right]$$

where

(13) $$\mathbf{rect}\frac{t-t_R}{T} = \begin{cases} 1, 0 \le \frac{t-t_R}{T} < 1, \\ 0, \frac{t-t_R}{T} < 0 \\ 0, \frac{t-t_R}{T} \ge 1 \end{cases}$$

Finite LFM wave
The mathematical model of a single finite LFM wave with unity amplitude is the complex exponential function

(14) $$\dot{s}(t,R) = \mathbf{rect}\frac{t-t_R}{T}\mathbf{exp}\left\{ j\begin{bmatrix} \omega(t-t_R) + \\ + b(t-t_R)^2 \end{bmatrix}\right\}$$

where

$$\mathbf{rect}\frac{t-t_R}{T} = \begin{cases} 1, 0 \le \frac{t-t_R}{T} < 1, \\ 0, \frac{t-t_R}{T} < 0 \\ 0, \frac{t-t_R}{T} \ge 1 \end{cases}$$ is the rectangular function.

e. Wave trains

Finite monochromatic wave train
The mathematical model of a finite monochromatic wave train with unity amplitude and time delay t_R is the finite complex exponential function

$$(15)\qquad \tilde{s}(t,R) = \sum_t \mathbf{rect}\frac{t-t_R}{T}\mathbf{exp}\left[j\omega(t-t_R)\right]$$

where $\tilde{t} = t \bmod T_p$ is the slow time, $t = \tilde{t} - pT_p$ is the fast time, T_p is the pulse repetition period, and p is the number of the emitted wave.

Finite LFM wave train
The mathematical model of a finite LFM wave train with unity amplitude and time delay t_R is the finite complex exponential function

$$(16)\qquad \tilde{s}(t,R) = \sum_t \mathbf{rect}\frac{t-t_R}{T}\mathbf{exp}\left\{ j\begin{bmatrix}\omega(t-t_R)+ \\ +b(t-t_R)^2\end{bmatrix}\right\}$$

where $\tilde{t} = t \bmod T_p$ is the slow time, $t = \tilde{t} - pT_p$ is the fast time, T_p is the pulse repetition period, and p is the number of the emitted wave.

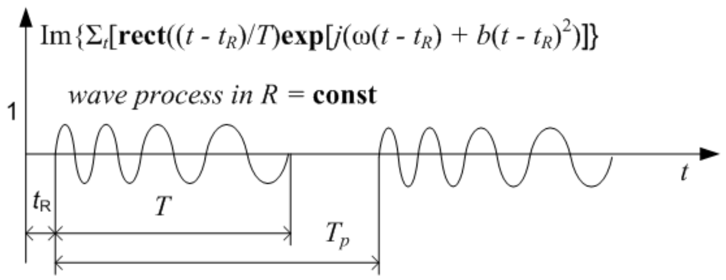

Fig. 6

The imaginary part of the finite LFM wave train with unity amplitude as a function of the time measured in a particular range distance R is presented in Fig. 6.

3. Signal formation models

a. Signal formation model with finite monochromatic waves
The radar emits electromagnetic wave trains to the object. The object is presented as an assembly of point scatterers placed on the reflecting object's surface. The radar receiver processes signals induced by waves reflected by object's point scatterers. The time delay from a particular point scatterer is proportional to $2R$, where R is the radar-object distance. If the object is moving trough radar pattern the distance R is a function of slow time $\tilde{t} = t + pT_p$. In general case the displacement of the object in the fast time t is negligible, then R is a function of pT_p, i.e. $R(pT_p)$ or $R(p)$. The monochromatic finite signal from a particular *ijk*th point scaterer with intensity a_{ijk} can be written as

(17) $$\tilde{s}_{ijk}(t)=\sum_p a_{ijk}\,\mathbf{rect}\frac{t-t_{ijk}(p)}{T}\,\mathbf{exp}\left\{j\left[\omega\left(t-t_{ijk}(p)\right)\right]\right\},\text{ where } t=\tilde{t}-pT_p.$$

The rectangular function is defined by the following expression

(18) $$\mathbf{rect}\frac{t-t_{ijk}}{T}=\begin{cases}0, \dfrac{t-t_{ijk}(p)}{T}\le 0\\ 1, 0<\dfrac{t-t_{ijk}(p)}{T}\le 1,\\ 0, \dfrac{t-t_{ijk}(p)}{T}>1\end{cases}$$

where

$t_{ijk}(p)=\dfrac{2R_{ijk}(p)}{c}$ denotes the time delay of the propagation of the wave from the radar to the *ijk*th point scatterer of the object and back to the radar; $R_{ijk}(p)$ denotes the distance measured from the radar to the *ijk*th point scatterer of the object.

The model of a monochromatic finite signal from the object is a geometrical sum of signals from all point scaterers and can be written as

(19) $$\tilde{s}(t)=\sum_p\sum_{ijk} a_{ijk}\,\mathbf{rect}\frac{t-t_{ijk}(p)}{T}\,\mathbf{exp}\left\{j\left[\omega\left(t-t_{ijk}(p)\right)\right]\right\},\text{ where } t=\tilde{t}-pT_p.$$

b. Signal formation model with finite LFM waves

The model of a LFM c finite signal from a particular *ijk*th point scaterer can be written as

(20) $$\tilde{s}_{ijk}(t)=\sum_p a_{ijk}\,\mathbf{rect}\frac{t-t_{ijk}(p)}{T}\,\mathbf{exp}\left\{j\begin{bmatrix}\omega\left(t-t_{ijk}(p)\right)+\\ +b\left(t-t_{ijk}(p)\right)^2\end{bmatrix}\right\},\text{ where } t=\tilde{t}-pT_p.$$

The model of a LFM finite signal from the object can be expressed as

(21) $$\tilde{s}(t)=\sum_p\sum_{ijk} a_{ijk}\,\mathbf{rect}\frac{t-t_{ijk}(p)}{T}\,\mathbf{exp}\left\{j\begin{bmatrix}\omega\left(t-t_{ijk}(p)\right)+\\ +b\left(t-t_{ijk}(p)\right)^2\end{bmatrix}\right\},\text{ where } t=\tilde{t}-pT_p.$$

4. Radar Imaging Geometry

The main geometrical characteristic is the distance from the radar to a particular point scatterer from the object space which can be defined be the following geometry. The radar scenario is depicted in 3-D coordinate system $0xyz$ (Fig. 7). The object, presented as an assembly of point scatterers is depicted in coordinate system $0'XYZ$.

The position of the radar is defined by distance vector $\mathbf{R}_r$. The current position of the *ijk*th point scatterer is defined by the following vector equation

(22) $\mathbf{R}_{ijk}(p) = \mathbf{R}_{r0}(p) + \mathbf{AR}_{ijk}$

where $\mathbf{R}_{r0}(p) = \mathbf{R}_r - \mathbf{R}_{00}(p)$ is the current vector distance from the radar to the origin of the coordinate system $0'XYZ$ of the object, $\mathbf{R}_{00}(p)$ is the current vector distance of from the origin of the coordinate system $0xyz$ to the origin of the coordinate system $0'XYZ$ of the object space.

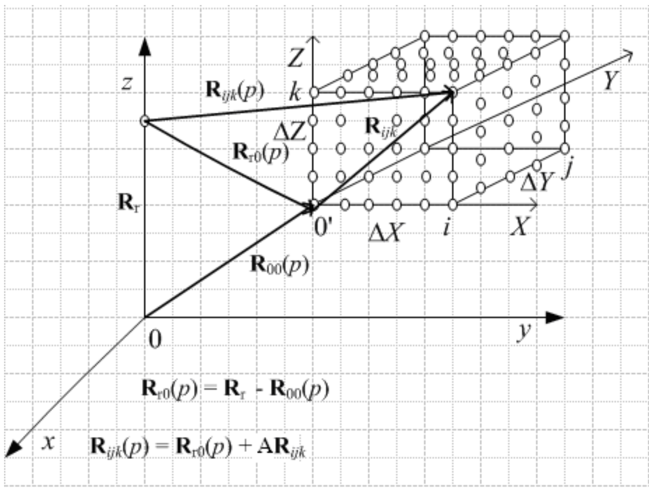

Fig. 7.

5. Conclusion

In the present work theoretical description of waveforms, waves and mathematical model of radar signal formation has been considered. Based on the complex exponential function continuous and finite waveforms, waves and radar signals have been analytically described and graphically illustrated. Radar imagining geometry has been presented and analytical described.

Acknowledgement

The work is supported by Project NATO CLG: ESP.EAP.CLG.983876, and Project ESA C1P-6051.

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