SAR MAPPING EQUATIONS AND COORDINATE TRANSFORMATIONS

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Abstract. This paper addresses basic SAR mapping equations, polynomial description of the satellite orbit. Range and Doppler equations, Earth ellipsoid equation and interferometric equation are defined in order to compute the coordinates of the target point placed on the Earth surface. Least mean square method for polynomial coefficient determination is developed. An algorithm for transformation between Earth-centered Earth-fixed (ECEF) coordinates and geodetic coordinates is presented.

Keywords: Coordinate Transformation, InSAR, Interferometry, SAR

INTRODUCTION

Synthetic Aperture Radar (SAR) is a coherent active microwave imaging system. SAR emits high informative pulses and records backscattered information from the target in a form of a complex signal carrying amplitude and phase information about reflective properties of an Earth surface. Interferometric SAR (InSAR) technique makes use of phase difference information extracted from two complex valued SAR images acquired from different orbit positions. This information can be used to determine geological and geophysical quantities such as topography of the Earth surface, slope, surface deformation (volcanoes, landslides, earthquakes) glaciers' movement studies, vegetation change etc. The importance of InSAR application is based on its high spatial resolution and good potential precision and capabilities for automated generation of Digital Elevation Model (DEM).

The image product from SAR is a picture of the Earth surface can be considered as a map with high resolution and scale accuracy. A profound problem arises in geolocation, i.e. determining the overall location of the image in geographic coordinates, namely geographic latitude longitude and altitude. The most effective method of geolocation is tiepointing in which known geographic features are matched between the image and map data obtained by conventional methods.

SAR being active radar instrument, provides precise information on the range from the satellite to the target and the Doppler history of a reflected by the target signal. These quantities can be related to precise determined satellite's coordinates and Earth surface coordinates. Therefore, it is possible to compose and solve a set of equations giving the Earth location for each image pixel to a high precision.

An automatic and precise orthorectification, co-registration, and sub-pixel correlation of satellite Images, application to ground deformation measurements is performed in [1]. The problems of interferometric SAR for DEM generation and validation of an integrated procedure based on multisource data are considered in [2, 3]. In InSAR processing simulation of interferogram is a common practice. It is used as synthetic data to test and validate the whole chain of InSAR processing from the interferogram creation to the DEM reconstruction. In [4] an algorithm to simulate the geocoding processing and phase to height conversion is suggested. The algorithm includes DEM of terrain simulation and interferogram simulation with DEM and satellite orbit parameters. A novel across-track SAR interferometry simulation for repeat-pass satellite InSAR studies are considered in [5,6,7,8].

The transformation between Earth-centered Earth-fixed (ECEF) coordinates and geodetic coordinates is required while processing SAR images in the cartography. There exist many papers devoted to this problem in early decades and a number of methods that can be divided into two categories: exact approaches and approximations have been proposed. Presently, there have been many different methods to approximate the exact transformation [9,10,11]. In [12] exact transformation formulas from Earth-fixed coordinates to geodetic coordinates are derived and compared with the approximation methods in complexity and in sensitivity to computer round–off error. In [13] an iterative method based on the Newton-Raphson (NR) approach is developed. This method exhibits efficiency and accuracy and is free from singularity and non-convergence except in a small region near the center of the Earth.

The main purpose of this work is twofold: first, definition of basic SAR mapping equations: range, Doppler and interferometric, and polynomial description SAR satellite orbit; second, description of an algorithm for transformation from Earth-Centered Earth-fixed coordinates to geodetic coordinates.

BASIC SAR MAPPING EQUATIONS

The coordinates of a target point fixed on the Earth surface can be determined by three SAR mapping equations: range equation, Doppler equation and Earth surface model equation if the height *h* of the target with respect to a reference Earth ellipsoid is known. Range equation is defined by the expression for the modulus of the slant range distance between SAR position **S** and target point **P** [2, 3]:

$$(1) \qquad R_S = \mathrm{mod}(\mathbf{P} - \mathbf{S}),$$

where $\mathbf{P} = (X_P, Y_P, Z_P)^T$ is the target location vector, $\mathbf{S} = (X_S, Y_S, Z_S)^T$ is the satellite location (state) vector; $(.)^T$ is the transpose index. The range equation, describes a sphere of radius R_S, centered in $\mathbf{S}$.

The slant range distance R_S is related to the slant range coordinate (*r*) of the SAR image trough the equation

(2) $$R_S = R_{S0} + \Delta R(r-1)$$

where R_{S0} is the nearest slant range to the SAR image.

The Doppler equation is defined by the expression for Doppler centroid frequency [2,3]

(3) $$f_D = -\frac{2.\mathbf{R}_S \cdot \mathbf{V}_S}{\lambda R_S},$$

where λ is the radar wavelength, $\mathbf{V}_S$ is the relative satellite – target velocity vector. Considering a zero-Doppler focusing algorithm the Doppler equation can be written as

(4) $$(\mathbf{S}-\mathbf{P})\cdot \mathbf{V}_S = 0.$$

This equation, identifies a plane orthogonal to the vector $\mathbf{V}_S$.

SAR image formation comprises kinematical orbital parameters, SAR parameters, and SAR processing parameters. SAR data correspond to an area of about 100 x 100 Km^2.

The Earth model equation has the form [14]

(5) $$\frac{X^2+Y^2}{(R_E+h)^2}+\frac{Z^2}{R_P^2}=1$$

where R_E is the equatorial radius of the Earth equal to 6378137 m; *h* is the height of the target with respect to a reference Earth ellipsoid;

(6) $$R_P = (1-f)(R_E+h);$$

where $f = 1/298.257223563$ is the flattening factor of the reference ellipsoid.

Given a slant range R_S and Doppler centroid f_D which have been computed for each pixel by the SAR processor and the height *h* of the target with respect to a reference Earth ellipsoid, it is possible to solve the system of equations (1), (3), (5) with respect to Earth location target point $\mathbf{P} = (X_P, Y_P, Z_P)^T$ by an iterative numerical technique (the system is non-linear).

In InSAR topographic applications, the height of pixel, *h*, is unknown and eq. (5) cannot be applied to obtain $\mathbf{P} = [X_P, Y_P, Z_P]$. In this case target coordinates can be derived using as a third constrain to SAR mapping equations (1) and (3) the interferometric phase equation defined for particular $\mathbf{P} = [X_P, Y_P, Z_P]$ from the interferogram based on SLC images of both master SAR and slave SAR.

In order to convert the interferometric phase field to terrain height relief a rigorous procedure transforming image space coordinates, azimuth, slant range and interferomeric phase to object space coordinates, $\mathbf{P} = [X_P, Y_P, Z_P]$, is applied. For each pixel with coordinates azimuth (*as*), range (*r*) from the interferogram, the object space coordinates, $\mathbf{P} = [X_P, Y_P, Z_P]$ are derived using two basic SAR mapping equations defined for the master SAR image and interferometric equation. The range and Doppler equations can be written as follows

(7) $R_S = \mathrm{mod}(\mathbf{S^M} - \mathbf{P})$ - range equation;

(8) $F_D = -\dfrac{2(\mathbf{S^M} - \mathbf{P}).\mathbf{V}_S}{\lambda\,\mathrm{mod}(\mathbf{S^M} - \mathbf{P})}$ - Doppler equation.

The interferometric equation is defined by the expression for the interferometric phase estimated by computing, for each target pixel **P** in the azimuth, slant range coordinate plane of acquisition, the difference in the SAR-target travel path distance for the two satellite positions $\mathbf{S^M}$ and $\mathbf{S^S}$.

(9) $\mathrm{mod}(\mathbf{S^S} - \mathbf{P}) = \mathrm{mod}(\mathbf{S^M} - \mathbf{P}) + \Phi(as, r)\,\dfrac{\lambda}{4\pi}$,

where $\mathbf{P} = [X_P, Y_P, Z_P]^T$ is the vector of unknown coordinates, $\Phi(as, r)$ is the interferometric phase defined in coordinates (*as*, *r*) of the unwrapped interferogram. The vector $\mathbf{S^M}$ and $\mathbf{S^S}$ for each pair coordinates (*as*, *r*) is defined by the expression for $\mathbf{S}(as)$.

This procedure of the target point $\mathbf{P} = [X_P, Y_P, Z_P]^T$ determination is performed for each pixel of the unwrapped interferogram, thus an irregular 3-D grid of pixels is obtained. The coordinates of the pixels are defined in the geocentric Cartesian system. Therefore, a transformation to a cartographic system and to optometric heights has to be performed.

POLYNOMIAL MODEL FOR THE SATELLITE ORBIT DESCRIPTION

In order to be solved non-linear system of equations (7, 8, 9) in respect of target point $\mathbf{P} = [X_P, Y_P, Z_P]^T$ the state location vector of satellites has to be expressed by azimuth coordinates of the SAR image (*as*, *r*). Even considering full-frame processing, a satellite trajectory can be well approximated by a low-order polynomial function. More precisely, for a third-order fitting, satellite position is defined by the following vector polynomial equation for the satellite location (state) vector **S**:

(10) $$\mathbf{S} = \mathbf{a}t_{as} + \mathbf{b}t_{as}{}^2 + \mathbf{c}t_{as}{}^3 + \mathbf{d} ,$$

where vectors {**a**, **b**, **c**, **d**} can be obtained by means of an LMS adjustment using a priory known satellite location (state) vectors whose coordinates are available in the image header file; $t_{as} = t - t_0$ is the time azimuth parameter, *t* is the time of acquisition, t_0 is the acquisition time of the first image azimuth.

For a particular target point $\mathbf{P} = (X_P, Y_P, Z_P)^T$ the time *t* is related to the azimuth coordinate (*as*) of the SAR image trough the equation [2, 3]

(11) $$t = t_0 + \Delta T(as - 1) ,$$

where ΔT is the time pixel size in azimuth direction; (*as*) is the index of SAR image in azimuth direction.

Thus the azimuth parameters is defined by $t_{as} = \Delta T(as - 1)$. The coordinates of the satellite state vector $\mathbf{S} = (X_S, Y_S, Z_S)^T$ and satellite vector velocity $\mathbf{V}_S$ can be expressed as functions of the image azimuths.

LMS determination of polynomial coefficients

In order to define two SAR mapping equations and one interferometric equation in image space coordinates (*as*, *r*) the satellite orbit coordinates in particular time of acquisition related to the azimuth coordinates of the SAR image, a polynomial interpolation of vector coordinates $\mathbf{S} = [X_S, Y_S, Z_S]^T$ is applied.

Let $\mathbf{S}(p) = [X_S(p), Y_S(p), Z_S(p)]^T$ be the vector measurements of satellite coordinates, p is the number of the emitted pulse (the number of the measurement). Let

$\mathbf{S}(p,\boldsymbol{\lambda}) = [X_S(p,\mathbf{a}), Y_S(p,\mathbf{b}), Z_S(p,\mathbf{c})]^T$ be the vector of third order polynomials, which in matrix form can be written as

(12) $$\mathbf{S}(p,\boldsymbol{\lambda}) = \begin{bmatrix} X_S(p,\mathbf{a}) \\ Y_S(p,\mathbf{b}) \\ Z_S(p,\mathbf{c}) \end{bmatrix} = \begin{bmatrix} a_0 & a_0 & a_2 & a_3 \\ b_0 & b_1 & b_2 & b_3 \\ c_0 & c_1 & c_2 & c_3 \end{bmatrix} . \begin{bmatrix} 1 \\ t_p \\ t_p^2 \\ t_p^3 \end{bmatrix},$$

where $t_p = p(\Delta T) - t_0$ is the time parameter, $p = \overline{N_0, N}$, $N_0 = \mathrm{int}\left(\frac{t_0}{\Delta T}\right)$ is the pulse number for acquisition of the first SAR image line, t_0 is the acquisition time of the first SAR image line, $\boldsymbol{\lambda} = [\mathbf{a}, \mathbf{b}, \mathbf{c}]^T$, $\mathbf{a} = [a_0, a_1, a_2, a_3]^T$, $\mathbf{b} = [b_0, b_1, b_2, b_3]^T$ and $\mathbf{c} = [c_0, c_1, c_2, c_3]^T$ are vectors of estimated coefficients. Vector coefficients can be derived by the method of least mean square (LMS) errors. Generalized iteration procedure to estimating vector polynomial parameters includes:

1. Computing the vector parameters $\boldsymbol{\lambda}_r$ on *r*-th iteration

(13) $$\boldsymbol{\lambda}_r = \boldsymbol{\lambda}_{r-1} + \boldsymbol{\Delta}_{\boldsymbol{\lambda}}$$

2. Computing vector of LMS errors $\boldsymbol{\Delta}_{\boldsymbol{\lambda}}$

(14) $$\boldsymbol{\Delta}_{\boldsymbol{\lambda}} = \left[\mathbf{B}^T.\mathbf{D}.\mathbf{B}\right]^{-1}\mathbf{B}^T.\mathbf{D}.\boldsymbol{\Delta}_{\mathbf{S}},$$

where $\boldsymbol{\Delta}_{\mathbf{S}} = \mathbf{S}(p) - \mathbf{S}(p,\boldsymbol{\lambda}) = [\Delta_{X_S}, \Delta_{Y_S}, \Delta_{Z_S}]^T$ is the vector measurements errors, which in matrix form can be described by the following equation:

(15) $$\boldsymbol{\Delta}_{\mathbf{S}} = \begin{bmatrix} \Delta_{X_S} \\ \Delta_{Y_S} \\ \Delta_{Z_S} \end{bmatrix} = \begin{bmatrix} X_S(p) - X_S(p,\mathbf{a}) \\ Y_S(p) - Y_S(p,\mathbf{b}) \\ Z_S(p) - Z_S(p,\mathbf{c}) \end{bmatrix}.$$

Elements of the matrix $\hat{\mathbf{B}} = \frac{\partial \mathbf{S}(p,\boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}}$ are coefficients of first order Taylor expansion. For example if $\boldsymbol{\lambda} = \mathbf{a}$, then the matrix $\mathbf{B} = \frac{\partial X_S(p,\mathbf{a})}{\partial \mathbf{a}}$ of dimensions [N-N_0, 4]. Elements B_{pi} of the matrix **B** can be expressed as $B_{pi} = \frac{\partial X_S(p, a_i)}{\partial a_i}$, where $p = \overline{N_0, N}$, $i = \overline{0,3}$.

For $i = 0 : B_{p0} = \frac{\partial X_S(p, a_i)}{\partial a_0} = 1$.

For $i = 1$, $B_{p1} = \frac{\partial X_S(p, a_i)}{\partial a_1} = t_p$.

For $i = 2$, $B_{p2} = \frac{\partial X_S(p, a_i)}{\partial a_2} = t_p^2$.

For $i = 3$, $B_{p3} = \frac{\partial X_S(p, a_i)}{\partial a_3} = t_p^3$.

Matrix **D** of dimensions [N-N_0, N-N_0] is diagonal with elements reciprocal to the dispersions of measurements.

Once the vector $\boldsymbol{\lambda}$ of coefficients has been defined for the area of observation polynomial description of the vector satellite coordinates **S** as a function of an azimuth image pixel's coordinate (*as*) can be written as

(16) $$\mathbf{S}(as) = \begin{bmatrix} X_S(as) \\ Y_S(as) \\ Z_S(as) \end{bmatrix} = \begin{bmatrix} a_0 & a_0 & a_2 & a_3 \\ b_0 & b_1 & b_2 & b_3 \\ c_0 & c_1 & c_2 & c_3 \end{bmatrix} . \begin{bmatrix} 1 \\ t_{as} \\ t_{as}^2 \\ t_{as}^3 \end{bmatrix},$$

where $t_{as} = t - t_0$, t is the acquisition time related to the azimuth coordinate (*as*) of the SAR image by $t = t_0 + \Delta T(as - 1)$, then $t_{as} = \Delta T(as - 1)$, where ΔT is the azimuth time pixel's size.

The described above procedure is applied to compute azimuth coordinates of both master SAR $\mathbf{S^M}$ and slave SAR $\mathbf{S^S}$ satellites.

Co-registration problem

In order to obtain exact values of target point's coordinates proper space alignment between the two SAR images (master and slave) should be performed on a pixel by pixel basis, with accuracy of the order of one tenth of the resolution cell size, or better. Co-registration mapping of the slave to master images can be performed by a polynomial of a second order that approximates the pixel-to-pixel displacement. It is assumed that the targets lie on an ellipsoidal Earth surface. In satellite-borne Synthetic Aperture Radars such as ERS and Envisat ASAR, the sensor velocity and attitudes are so stable that the master-slave deformation of an entire frame (100 × 100 km) can be well approximated by the following polynomial [14]:

(17)
$$r^S = a\left(r^M\right)^2 + br^M + c(as)^M + d$$
$$(as)^S = e\left(r^M\right)^2 + fr^M + g(as)^M + h,$$

where r^M, $(as)^M$ are range and azimuth coordinates of the master SAR image; r^S, $(as)^S$ are range and azimuth coordinates of the slave SAR image.

Co-registration coefficients can be computed by a least mean square regression based on a regular grid of points displaced over the whole frame of the SAR image.

In order the master SAR satellite $\mathbf{S^M}$, slave SAR satellite $\mathbf{S^S}$ and SAR image pixel **P** to lie in the same Doppler centroid plane, vector coordinates of the position of slave SAR satellite $\mathbf{S^S}$ has to satisfy the following equation

(18)
$$\left(\mathbf{S^M} - \mathbf{S^S}\right)\mathbf{V^M} = -\lambda \frac{F_D}{2} \operatorname{mod}\left(\mathbf{S^M} - \mathbf{S^S}\right).$$

Denote $\mathbf{B} = \left(\mathbf{S^M} - \mathbf{S^S}\right)$ and $B = \operatorname{mod}\left(\mathbf{S^M} - \mathbf{S^S}\right)$ vector baseline and baseline length (slave to master distance). For each azimuth coordinate $(as)^M$ from the master image, the azimuth coordinate $(as)^S$ for the slave image is computed.

TRANSFORMATIONS FROM EARTH-CENTERED EARTH-FIXED COORDINATES TO GEODETIC COORDINATES

The transformation from GEI to geodetic coordinate system can be performed by algorithm described in [13]:

1. Given coordinates of the target point vector $\mathbf{P} = \left[X_P, Y_P, Z_P\right]$, Earth ellipsoid parameters length of the semimajor axis R (R = 6378.137 km), length of semiminor axis r (r = 6356.7523142 km), flatteninig of the ellipsoid f ($f = \frac{R-r}{R}$), iterative accuracy Δ, compute coordinates x_0, z_0 by expression

(19)
$$x_0 = \left(X_P^2 + Y_P^2\right)^{\frac{1}{2}}, \; z_0 = Z_P.$$

2. Compute coefficients of the quartic order polynomial $f(t)$

(20)
$$f(t) = At^4 + (B+C)t^3 + (B-C)t - A = 0,$$

(21)
$$A = rz_0, \; B = 2Rx_0, \; C = 2\left(R^2 - r^2\right).$$

Iteration procedure

3. Compute geodetic longitude λ by expression

(22) $\lambda = \arctan 2(Y_P, X_P)$

4. Compute the parameter t_0 , initial guess for the variable t of the quartic order polynomial

(23) $$t_0 = (f-1)\frac{x_0}{z_0} + \mathrm{sgn}(z_0)\left\{1 + \left[(f-1)\right]^2 \frac{x_0}{z_0}\right\}^{\frac{1}{2}}$$

where $\mathrm{sgn}(x) = \begin{cases} +1 if\ x > 0 \\ -1 if\ x < 0 \end{cases}$.

5. Compute next value of the parameter t by

(24) $$t_{k+1} = t_k - \frac{f(t_k)}{f'(t_k)},$$

where $f'(t_k)$ is the first derivation of $f(t)$ for $t = t_k$.

6. If $|t_k - t_{k-1}| \le \Delta$ compute geodetic latitude L by

(25) $$L = \mathrm{arc\,cot}\left[(f-1)\frac{t^2-1}{2t}\right].$$

7. Compute the geodetic height

(26) $$h = \mathrm{sgn}(L)\left(z_0 - r\frac{2t}{1+t^2}\right)\left\{1 + \left[(f-1)\right]^2 \frac{x_0}{z_0}\right\}^{\frac{1}{2}}.$$

The described algorithm exhibits good efficiency and accuracy. There not exist singularity and non-convergence for all points in space excluding a region near the center of the Earth of radius about 50 km.

CONCLUSIONS

In this paper basic SAR mapping equations, polynomial approximation of the satellite orbit have been described. Range and Doppler equations and Earth ellipsoid equation have been defined in order to compute the coordinates of the target point placed on the Earth surface. Interpolation polynomial of third order for satellite orbit has been described. Least mean square method for polynomial coefficient determination has been developed. An algorithm for transformation from geocentric coordinates to geodetic coordinates has been considered

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