ISAR SIGNAL FORMATION AND IMAGE EXTRACTION AS DIRECT AND INVERSE PROJECTIVE OPERATION

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Abstract: *In this work ISAR (Inverse Synthetic Aperture Radar) signal formation and image reconstruction are analytically described and interpreted as direct and inverse projection operation. LFM (Liner Frequency Modulated) and PCM (Phase Code Modulated) emitted waveforms are used while modeling reflected ISAR returns. Complex projection operator and its Taylor expansion in the signal formation and image reconstruction procedures are defined.*

Key words: *ISAR, LFM, PCM, ISAR signal model, ISAR image reconstruction*

1. Introduction

Inverse Synthetic Aperture Radar (ISAR) technique enjoys intensive research activities over the last twenty years. It makes an impact on the progress in synthetic aperture radar technologies and meets strong requirements for the further enhancement of microwave remote sensing systems. The implementation of ISAR concept will enlarge the aria of application and improve substantially the functionality of imaging radars and moving target recognition.

ISAR technique is an efficient approach to obtain high quality images of moving objects. The image extraction consists in coherent processing of ISAR signal returns, received during relative motion of the target with respect to the ISAR system, placed in the origin of the coordinate system of observation. High range resolution of the image can be achieved by ISAR transmitted signals with a large bandwidth. High azimuth resolution can be realized using large synthetic aperture length during relative motion of the object in respect of the ISAR system of observation.

A classical ISAR image reconstruction technique is a range-Doppler compression accompanying with phase compensation [1,2]. Based on Peter Stoica's spectral models of ISAR signals powerful computational image reconstruction methods as parametric and semi-parametric methods are developed in [3,4]. Joint time-frequency transform for radar range-Doppler imaging and ISAR motion compensation via adaptive joint time-frequency technique is presented in [5-7].

Despite detailed description of imaging techniques in literature it is necessary to emphasize that in its essence the image reconstruction is phase compensation, i.e. removing all signal phases induced by motion of the target. Moreover the signal formation and image extraction can be considered as direct and inverse projections with one and the same operator. This paper is written in response to this problem. The main goal of the paper is to define signal formation and image extraction both in the field of LFM and PCM signal plane and prove that these operations can be interpreted as direct and inverse space transformations.

The remainder of the paper is organized as follows. In Section II LFM waveform and ISAR signal formation and image reconstruction are defined. In Section III PCM waveform and ISAR signal formation and image reconstruction are defined. In Section IV phase compensation technique is described. In Section V results of numerical experiments and their interpretation are given.

2. LFM waveforms and ISAR signal model

LFM waveforms

The ISAR emits to the moving target a series of linear frequency modulated waveforms, each of which is described by

$$\dot{S}(t) = \mathbf{rect}\frac{t}{T}\mathbf{exp}\left\{-j\left[\omega t + bt^2\right]\right\} \tag{1}$$

where $\omega = 2\pi\frac{c}{\lambda}$ is the angular frequency; $c = 3.10^8$ m/s is the speed of the light; λ is the wavelength of the signal; T is the time duration of a LFM pulse; $b = \frac{2\pi\Delta F}{T}$ is the LFM rate. The bandwidth $(2\Delta F)$ of the transmitted pulse provides the dimension of the range resolution cell $\Delta R = c/2\Delta F$.

LFM ISAR signal model

The ISAR signal is a geometrical sum of signals reflected from each point scatterer of the object. The deterministic component of the ISAR signal, reflected by ijkth point scatterer of the target can be described by the expression

$$\dot{S}_{ijk}(p,t) = a_{ijk}\,\mathbf{rect}\frac{t - t_{ijk}(p)}{T}\,\mathbf{exp}\left\{-j\left[\omega\left(t - t_{ijk}(p)\right) + b\left(t - t_{ijk}(p)\right)^2\right]\right\} \tag{2}$$

where $\mathbf{rect}\frac{t - t_{ijk}(p)}{T} = \begin{cases} 1, 0 \le \frac{t - t_{ijk}(p)}{T} < 1, \\ 0, \frac{t - t_{ijk}(p)}{T} < 0 \\ 0, \frac{t - t_{ijk}(p)}{T} \ge 1 \end{cases}$

where a_{ijk} is the reflection coefficient of the ijkth point scatterer, a 3-D image function; $t_{ijk}(p) = \frac{R_{ijk}(p)}{c}$ is the time delay of the signal from the ijkth point scatterer; t is the time dwell or the fast time of the ISAR signal which in discrete form can be written as $t = [k_{ijk\,\min}(p) + k - 1]\Delta T$, where $k = \overline{1, [k_{ijk\,\max}(p) - k_{ijk\,\min}(p)] + K}$ is the sample number of a LFM pulse; $K = \frac{T}{\Delta T}$ is the full number of samples of the LFM pulse, ΔT is the time duration of a LFM sample, $k_{ijk\,\min}(p) = \left\lceil \frac{t_{ijk\,\min}(p)}{\Delta T} \right\rceil$ is the number of the radar range bin where the signal, reflected by the nearest point scatterer of the target is detected, $t_{ijk\,\min}(p) = \frac{R_{ijk\,\min}(p)}{c}$ is the minimal time delay of the ISAR signal reflected from the nearest point scatterer of the target, $K(p) = k_{ijk\,\max}(p) - k_{ijk\,\min}(p)$ is the relative time dimension of the target; $k_{ijk\,\max}(p) = \left\lceil \frac{t_{ijk\,\max}(p)}{\Delta T} \right\rceil$ is the number of the radar range bin where the signal, reflected by farthest point scatterer of the target is detected; $t_{ijk\,\max}(p) = \frac{R_{ijk\,\max}(p)}{c}$ is the maximum time delay of the ISAR signal reflected from the farthest point scatterer of the target; $R_{ijk}(p)$ is the module of the range distance vector to the ijk-th point scatterer of the target.

The deterministic components of the ISAR signal return from the target are defined as a superposition of signals reflected by point scatterers placed on the target, i.e.

$$\begin{aligned}\dot{S}(p,t) &= \sum_{ijk} \dot{S}_{ijk}(p,t) \\ &= \sum_{ijk} a_{ijk}\,\mathbf{rect}\frac{t - t_{ijk}(p)}{T}\,\mathbf{exp}\left\{-j\left[\omega\left(t - t_{ijk}(p)\right) + b\left(t - t_{ijk}(p)\right)^2\right]\right\}\end{aligned} \tag{3}$$

which in discrete form can be written as

$$\begin{aligned}\dot{S}(p,t) &= \sum_{ijk} \dot{S}_{ijk}(p,t) \\ &= \sum_{ijk} a_{ijk}\,\mathbf{rect}\frac{(k_{ijk\,\min}(p) + k)\Delta T - t_{ijk}(p)}{T} \times \mathbf{exp}\left\{-j\begin{bmatrix}\omega\left((k_{ijk\,\min}(p) + k)\Delta T) - t_{ijk}(p)\right) + \\ + b\left((k_{ijk\,\min}(p) + k)\Delta T) - t_{ijk}(p)\right)^2\end{bmatrix}\right\}\end{aligned} \tag{4}$$

where

$$\begin{aligned}&\mathbf{rect}\frac{(k_{ijk\,\min}(p) + k)\Delta T - t_{ijk}(p)}{T} \\ &= \begin{cases} 1 \text{ if } 0 \le \frac{(k_{ijk\,\min}(p) + k)\Delta T - t_{ijk}(p)}{T} < 1 \\ 0 \text{ if } \frac{(k_{ijk\,\min}(p) + k)\Delta T - t_{ijk}(p)}{T} < 0 \\ 0 \text{ if } \frac{(k_{ijk\,\min}(p) + k)\Delta T - t_{ijk}(p)}{T} \ge 1 \end{cases}\end{aligned} \tag{5}$$

where $k = \overline{0, [k_{ijk\,\max}(p) - k_{ijk\,\min}(p)] + K - 1}$.

The procedure of ISAR signal formation is graphically illustrated in Fig. 1. Each LFM ISAR signal is presented by a sequence of seven samples which are placed in seven range bins with different numbers. Signals from nearest, first, second and third point scatterers are summed in the range bin $k_{ijk\,\min}$.

In the next range bin signals from the nearest, first, second, third and fourth point scatterers are summed. Finally, signals from third and fourth point scatterers are summed in the eight range bin.

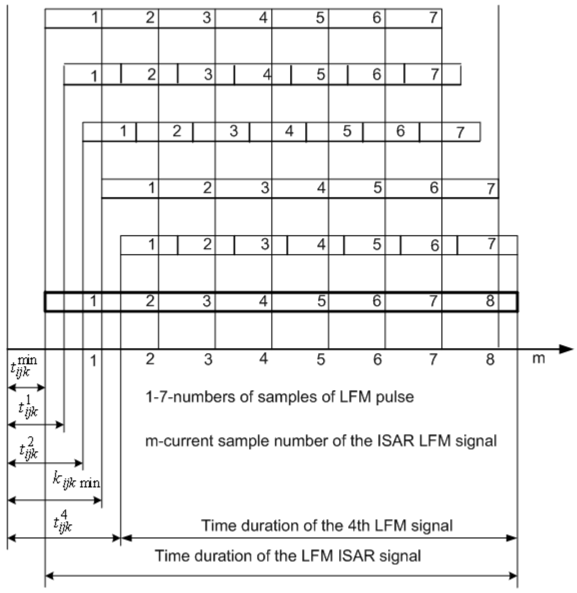


Figure 1. LFM ISAR signal formation

Demodulation (dechirping) of the ISAR signal return is performed by multiplication with a complex conjugated emitted waveform, i.e.

$$\hat{S}(p,t) = S(p,t) \times \mathbf{rect}\frac{t}{T}\exp\left[-j(\omega t + bt^2)\right]$$
$$= \sum_{ijk} a_{ijk}\,\mathbf{rect}\frac{t - t_{ijk}(p)}{T}\exp\left\{j\left[\omega\left(t - t_{ijk}(p)\right) + b.\left(t - t_{ijk}(p)\right)^2\right]\right\}.\exp\left[-j(\omega t + bt^2)\right] \quad (6)$$

which yields

$$\hat{S}(p,t) = \sum_{ijk} a_{ijk}\,\mathbf{rect}\frac{t - t_{ijk}(p)}{T}\exp\left\{-j\left[(\omega + 2bt)t_{ijk}(p) - bt_{ijk}^2(p)\right]\right\} \quad . \quad (7)$$

Denote the current angular frequency of emitted LFM pulse as $\omega(t) = \omega + 2bt$, where ω is the carrier angular frequency, and b is the chirp rate, $t = k\Delta T$ is the discrete time parameter, where k is the sample number, ΔT is the sample time duration. Then the current discrete frequency can be written as $\omega_k = \omega + 2b(k\Delta T)$ or $\omega_k = k\left(\frac{\omega}{k} + 2b(\Delta T)\right) = k\Delta\omega_k$.

Then expression (7) can be rewritten as

$$\hat{S}(p,t) = \sum_{ijk} a_{ijk}\,\mathbf{rect}\frac{t - t_{ijk}(p)}{T}\exp\left[-j\left(2\omega(t)\frac{R_{ijk}(p)}{c} - b\left(\frac{2R_{ijk}(p)}{c}\right)^2\right)\right] \quad (8)$$

which in discrete form can be written as

$$\hat{S}(p,k) = \sum_{ijk} a_{ijk}\,\mathbf{rect}\frac{(k_{ijk\,\min}(p) + k)\Delta T - t_{ijk}(p)}{T}\exp\left[-j\left(2\omega_k\frac{R_{ijk}(p)}{c} - b\left(\frac{2R_{ijk}(p)}{c}\right)^2\right)\right] \quad (9)$$

The expression (9) can be interpreted as a projection of the three-dimensional image function a_{ijk} onto two-dimensional ISAR signal plane $\hat{S}(p,k)$ by the projective operator, the exponential term

$$\exp\left[-j\left(2\omega_k \frac{R_{ijk}(p)}{c} - b\left(\frac{2R_{ijk}(p)}{c}\right)^2\right)\right]. \tag{10}$$

Formally the 3-D image function, a_{ijk} can be extracted from 2-D ISAR signal plane by the inverse operation

$$a_{ijk} = \sum_{p=1}^{N}\sum_{k=1}^{K}\hat{S}(p,k).\exp\left[j\left(2\omega_k \frac{R_{ijk}(p)}{c} - b\left(\frac{2R_{ijk}(p)}{c}\right)^2\right)\right], \tag{11}$$

where k is the discrete coordinate measured onto the line of sight of the object's geometric center, p is the discrete coordinate measured onto orthogonal direction to the line of sight of the object's geometric center.

Consequently, the extraction of the image function is a procedure of complete phase compensation of the signals reflected by all point scatterers from the object that means total compensation of target movement during inverse aperture synthesis. Through transformation of the discrete object's coordinates (*ijk*) into projective coordinates in (*k*, *p*) plane at the time instant of target imaging, and mathematical manipulations, the argument of the exponential term, $\left(2\omega_k \frac{R_{ijk}(p)}{c} - b\left(\frac{2R_{ijk}(p)}{c}\right)^2\right)$, the complex function that is infinitely differentiable in a neighborhood of projective coordinates can be expanded in Taylor series, the polynomial of higher order, i.e.

$$\left(2\omega_k \frac{R_{ijk}(p)}{c} - b\left(\frac{2R_{ijk}(p)}{c}\right)^2\right) == a_0 + a_1.(pT_p) + a_2.(pT_p)^2 + ... + a_m(pT_p)^m \tag{12}$$

$$+ b_1.(k\Delta T) + b_2.(k\Delta T)^2 + ... + b_m(k\Delta T)^m.$$

The constant terms a_0 has nothing to do with the phase correction and can be assumed zero. The linear terms $a_1.(pT_p)$ and $b_1.(k\Delta T)$ can be redefined as $a_1.(pT_p) = 2\pi \frac{p\hat{p}}{N}$ and $b_1.(k\Delta T) = 2\pi \frac{k\hat{k}}{K}$, where $\hat{k} = \hat{k}(ijk), \hat{p} = \hat{p}(ijk)$ are the new unknown coordinates of the *ijk*-th point scatterer on the plane (*p*, *k*) at the time instant of imaging.

Denote $\Phi(p,k) = a_2.(pT_p)^2 + ... + a_m(pT_p)^m + b_2.(k\Delta T)^2 + ... + b_m(k\Delta T)^m$, where the quadratic terms $a_2.(pT_p)^2$ and $b_2.(k\Delta T)^2$ perform quadratic phase correction, whereas the higher order terms denoted as $a_m pT_p)^m$ and $b_m(k\Delta T)^m$ perform higher order phase correction, then

$$\left(2\omega_k \frac{R_{ijk}(p)}{c} - b\left(\frac{2R_{ijk}(p)}{c}\right)^2\right) = 2\pi \frac{p\hat{p}}{N} + 2\pi \frac{k\hat{k}}{K} + \Phi(p,k) \tag{13}$$

The substitution of the expression (18) in the expression (16) and projection of the three dimensional image function a_{ijk} onto two-dimensional one $a(\hat{p},\hat{k})$ yield

$$a_{ijk}(\hat{p},\hat{k}) = \sum_{p=1}^{N}\sum_{k=1}^{K}\hat{S}(p,k)\exp\left[j\left(\Phi(k,p)+2\pi\frac{p\hat{p}}{N}+2\pi\frac{k\hat{k}}{K}\right)\right] \tag{14}$$

which can be rewritten as

$$a_{ijk}(\hat{p},\hat{k}) = \sum_{p=1}^{N}\left[\sum_{k=1}^{K}\hat{S}(p,k).\exp[j\Phi(k,p)].\exp\left(j2\pi\frac{k\hat{k}}{K}\right)\right]\exp\left(j2\pi\frac{p\hat{p}}{N}\right). \tag{15}$$

The expression (20) can be considered as an image reconstruction procedure, which does reveal a 2-D discrete complex image function $a_{ijk}(\hat{p},\hat{k})$.

Image Reconstruction Algorithm

The resulting signal carries information regarding two-dimensional (2-D) geometry of the target. Delays of the signals reflected from the point scatterers of the object is referred to as range dimension and can be extracted by standard fast Fourier transformation called range compression. Doppler shifts of the received signal are referred to as azimuth dimension and can be extracted by standard fast Fourier procedure as well applied over the range compressed signal.

Basic Operations

Image reconstruction can be implemented by following basic computational operations.

First, phase correction of the complex matrix $\hat{S}(p,k)$ by multiplication with complex exponential term $\exp[j\Phi(p,k)]$, i.e.

$$\tilde{S}(p,k) = \hat{S}(p,k).\exp[j\Phi(p,k)] \tag{16}$$

Second, range compression by discrete inverse Fourier transform of the phase corrected complex matrix $\tilde{S}(p,k)$ along the range coordinate k, i.e.

$$\tilde{S}(p,\hat{k}) = \frac{1}{K}\sum_{k=1}^{K}\tilde{S}(p,k).\exp\left(j2\pi\frac{k\hat{k}}{K}\right). \tag{17}$$

Third, image extraction by discrete inverse Fourier transform of a range compressed complex matrix $\tilde{S}(p,\hat{k})$ on the cross range coordinate p, i.e.

$$a_{ijk}(\hat{p},\hat{k}) = \frac{1}{N}\sum_{p=1}^{N}\tilde{S}(p,\hat{k}).\exp\left(j2\pi\frac{p\hat{p}}{N}\right). \tag{18}$$

The aforementioned algorithm is feasible if the phase correction function $\Phi(p,k)$ is a priory known. Otherwise, it is impossible to perform the image extraction procedure. Taking into account linear property of computational operations in (15) the image extraction algorithm may start with two-dimensional discrete Fourier transformation (range and cross range compression) over the demodulated ISAR signal, the complex matrix $\hat{S}(p,k)$

$$a_{ijk}(\hat{p},\hat{k}) = \sum_{p=1}^{N}\left[\sum_{k=1}^{K}\hat{S}(p,k).\exp\left(j2\pi\frac{k\hat{k}}{K}\right)\right]\exp\left(j2\pi\frac{p\hat{p}}{N}\right). \tag{19}$$

$$\left|a_{ijk}(\hat{p},\hat{k})\right| = \left|\sum_{p=1}^{N}\left[\sum_{k=1}^{K}\hat{S}(p,k).\exp\left(j2\pi\frac{k\hat{k}}{K}\right)\right]\exp\left(j2\pi\frac{p\hat{p}}{N}\right)\right| \quad (20)$$

3. Phase Code Modulated Waveform and ISAR Signal Model

Phase Code Modulated Waveform

During inverse aperture synthesis ISAR illuminates the target by a sequence phase-code modulated pulse trains. Each phase-code modulated pulse train is described by the expression

$$S(t) = a.\mathbf{rect}\frac{t}{T}\exp\{-j[\omega(t) + \pi b(t) + \varphi_0]\} \quad (21)$$

where $\tilde{t} = t \bmod T_p$ is the slow time and $t = \tilde{t} - pT_p$ is the fast time; p is the index of emitted pulse; T_p is the pulse repetition period;

$$\mathbf{rect}\frac{t}{T} = \begin{cases} 1, \text{if } 0 \le \frac{t}{T} \le 1; \\ 0, \text{otherwise}; \end{cases}$$

a is the amplitude of the transmitted pulses, $\omega = 2\pi c/\lambda$ is the signal angular frequency, φ_0 is the initial phase of a PCM pulse, $t = k.\Delta T$ $k = \overline{1,K}$ is the index of the PCM segment, $K = \frac{T}{\Delta T} = 1023$ is the full number of segments of the phase-code modulated pulse train, T is the time duration of phase-code modulated pulse train, ΔT - the time duration of the phase segment, $b(t)$ is the binary parameter of phase-code modulated pulse train

Phase Code Modulated ISAR signal model

Three dimensional deterministic component of the ISAR signal return reflected by ijk-th point scatterer is defined by

$$S_{ijk}(p,t) = a_{ijk}\,\mathbf{rect}\frac{t - t_{ijk}(p)}{T}\,\mathbf{exp}\{-j[\omega(t - t_{ijk}(p)) + \pi b(t)]\} \quad (22)$$

$$\mathbf{rect}\frac{t - t_{ijk}(p)}{T} = \begin{cases} 1, \text{ if } 0 \le \frac{t - t_{ijk}(p)}{T} < 1; \\ 0, \text{ if } \frac{t - t_{ijk}(p)}{T} < 0 \text{ and } \frac{t - t_{ijk}(p)}{T} \ge 1; \end{cases} \quad (23)$$

a_{ijk} is the reflective coefficient of the ijkth point scatterer from the object space, 3-D image function; $t_{ijk}(p) = \frac{R_{ijk}(p)}{c}$ - is the time delay from the ijk th point scatterer. The time dwell t of the ISAR signal return for each transmitted pulse, p, acquires values in the interval: $t_{ijk\,\min}(p) \le t \le t_{ijk\,\max}(p) + T$, which in discrete form can be written as

$t = t_{ijk\,\min}(p) + (k-1)\Delta T$ as well, where $k = \overline{1, L+K-1}$, $L = \mathbf{int}\left[\frac{t_{ijk\,\max}(p) - t_{ijk\,\min}(p)}{\Delta T}\right]$ is the relative time dimension of the target, $t_{ijk\,\min}(p) = \frac{R_{ijk\,\min}(p)}{c}$ is the minimal time delay of the ISAR signal from the target $t_{ijk\,\max}(p) = \frac{R_{ijk\,\max}(p)}{c}$ is the maximum time delay of the ISAR signal return from the target, $k_{ijk\,\min}(p) = \text{int}\left[\frac{R_{ijk\,\min}(p)}{c\Delta T}\right]$ $k_{ijk,\min}(p) = \text{int}\left[\frac{R_{ijk\,\min}(p)}{c\Delta T}\right]$ is the minimum range bin number where the ISAR signal from nearest point scatterer is detected, is the minimum range bin, but only the number of range bin where the signal from nearest point scatterer is detected, $k_{ijk\,\max}(p) = \text{int}\left[\frac{R_{ijk\,\max}(p)}{c\Delta T}\right]$ is number where the ISAR signal from furthest point scatterer is detected. If the exact value of $t_{ijk,\min}(p)$ is unknown but only $k_{ijk\,\min}(p)$, then the discrete time can be expressed as $t = [k_{ijk\,\min}(p) + (k-1)]\Delta T$.

The deterministic components of the ISAR signal return, reflected by all point scatterers of the object for every pth pulse are derived by

$$S(p,t) = \sum_{ijk} a_{ijk} \,\mathbf{rect}\frac{t - t_{ijk}(p)}{T} \,\mathbf{exp}\left\{- j\left[\omega(t - t_{ijk}(p)) + \pi b(t)\right]\right\}. \tag{24}$$

In discrete form the equation (24) can be written as

$$\begin{aligned} S(p,k) = \sum_{ijk} a_{ijk} \,\mathbf{rect}&\frac{t_{ijk\,\min}(p) + (k-1)\Delta T - t^{r}_{ijk}(p)}{T} \\ &\times \mathbf{exp}\left\{- j\left[\omega(t_{ijk\,\min}(p) + (k-1)\Delta T - t^{r}_{ijk}(p)) + \pi b((k-r+1)\Delta T)\right]\right\} \end{aligned} \tag{25}$$

where

$$\mathbf{rect}\frac{t_{ijk,\min}(p) + (k-1)\Delta T - t^{r}_{ijk}(p)}{T} = \begin{cases} 1, \text{ if } 0 \le \dfrac{t_{ijk\,\min}(p) + (k-1)\Delta T - t^{r}_{ijk}(p)}{T} < 1; \\ 0, \text{ if } \dfrac{t_{ijk\,\min}(p) + (k-1)\Delta T - t^{r}_{ijk}(p)}{T} < 0; \\ 0, \text{ if } \dfrac{t_{ijk\,\min}(p) + (k-1)\Delta T - t^{r}_{ijk}(p)}{T} \ge 1. \end{cases}$$

After arrangement of the time delays, $t_{ijk}(p)$ in ascending order, the equation (25) can be written as

$$\begin{aligned} S(p,k) = \sum_{m} a_{m} \,\mathbf{rect}&\frac{t_1(p) + (k-1)\Delta T - t^{r}_{m}(p)}{T} \\ &\times \mathbf{exp}\left\{- j\left[\omega(t_1(p) + (k-1)\Delta T - t^{r}_{m}(p)) + \pi b((k-r+1)\Delta T)\right]\right\} \end{aligned} \tag{26}$$

where

$$\mathbf{rect}\frac{t_1(p)+(k-1)\Delta T-t_m^r(p)}{T}=\begin{cases}1, \text{ if } 0\le\frac{t_1(p)+(k-1)\Delta T-t_m^r(p)}{T}<1;\\ 0, \text{ if } \frac{t_1(p)+(k-1)\Delta T-t_m^r(p)}{T}<0;\\ 0, \text{ if } \frac{t_1(p)+(k-1)\Delta T-t_m^r(p)}{T}\ge 1.\end{cases}$$

where $t_1(p)=t_{ijk\min}(p)$, m is the current number of $t_{ijk}(p)$ in the ascending sequence, r denotes a current number k while the rectangular function for a particular $t_m(p)=t_{ijk}(p)$ accepts value 1 first time. It is possible for many time delays, $t_{ijk}(p)$ the index r to have one and the same value. The index r can be considered as a projective discrete coordinate of *ijk*-th point scatterer on the range direction

ISAR Image Reconstruction Procedure

The phase demodulated ISAR signal has the form

$$\hat{S}(p,k)=\sum_{ijk} a_{ijk}\,\mathbf{rect}\frac{t_{ijk\min}(p)+(k-1)\Delta T-t_{ijk}^r(p)}{T}\times\mathbf{exp}\left\{-j\left[\omega(t_{ijk\min}(p)-t_{ijk}^r(p))+\pi b((k-r+1)\Delta T)\right]\right\} \quad (27)$$

After phase demodulation the 2-D projection of the 3-D image function, $a_{ijk}(\hat{p},r)$ on the range and cross range direction can be extracted by an inverse operation, i.e.

$$a_{ijk}(\hat{p},r)=\sum_{p=1}^{N}\sum_{k=r}^{r+K-1}\hat{S}(p,k)\mathbf{exp}\left\{j\left[\omega(t_{ijk\min}(p)-t_{ijk}^r(p))+\pi b((k-r+1)\Delta T)\right]\right\}, \quad (28)$$

where $a_{ijk}(\hat{p},r)$ is the 2-D projection of the 3-D image function on cross range ($\hat{p}$) and range (r) coordinates

Rename the range parameter r as a range coordinate $\hat{k}$ and take into account the linearity of the summation operation in (28) the expression (28) can be written as

$$a_{ijk}(\hat{p},\hat{k})=\sum_{p=1,N}\sum_{k=\hat{k}}^{\hat{k}+K-1}\left[\hat{S}(p,k)\exp j[\pi b((k-\hat{k}+1)\Delta T)\right]\mathbf{exp}\left\{j\omega\left[t_{ijk\min}(p)-t_{ijk}^{\hat{k}}(p)\right]\right\}. \quad (29)$$

Taylor expansion of the phase term $\omega[t_{ijk\min}(p)-t_{ijk}^{\hat{k}}(p)]$ can be presented as a polynomial function of higher order, i.e.

$$\omega[t_{ijk\min}(p)-t_{ijk}^{\hat{k}}(p)]=a_0+a_1(pT_p)+a_2(pT_p)^2+...+a_m(pT_p)^m. \quad (30)$$

The linear term $a_1(pT_p)$ can be reduced to $\frac{2\pi}{N}\hat{p}.p$ and considered as a Fourier operator, $\hat{p}$ is the discrete coordinates of *ijk*-th point scatterer placed in the *k*-th range bin, N is the number of emitted Barker's trains during the aperture synthesis. The constant term has nothing to do with the image reconstruction and can be removed. Denote $\Phi(p)=a_2(pT_p)^2+...+a_m(pT_p)^m$, then the equation (16) can be expressed as

$$a_{ijk}(\hat{p},\hat{k}) = \sum_{p=\overline{1,N}} \sum_{k=\hat{k}}^{\hat{k}+K-1} \left[\hat{S}(p,k)\exp j[\pi b((k-\hat{k}+1)\Delta T)\right]\mathbf{exp}\left\{j\left[\frac{2\pi}{N}\hat{p}p + \Phi(p)\right]\right\}. \quad (31)$$

Based on linearity of the operations in (31) it follows

$$a_{ijk}(\hat{p},\hat{k}) = \sum_{p=\overline{1,N}} \left\{ \sum_{k=\hat{k}}^{\hat{k}+K-1} \left[\hat{S}(p,k).\mathbf{exp}(j\Phi(p)\right]\mathbf{exp}[j\pi b((k-\hat{k}+1)\Delta T)]\right\}\mathbf{exp}\left\{j\left[\frac{2\pi}{N}\hat{p}p\right]\right\}. \quad (32)$$

Accordingly, image extraction procedure is accomplished in following stages: phase demodulation of the ISAR signal, distribution of the signal in range bins and range alignment, phase correction, range compression, azimuth compression.

a. Range alignment is accomplished along with the signal registration by renumbering the range cells, where the ISAR signal return is placed.

b. Phase correction is accomplished by multiplication of the phase demodulated ISAR signal with an exponential phase correction function, i.e.

$$\tilde{S}(p,k) = \hat{S}(p,k).\mathbf{exp}(j\Phi(p) \quad (33)$$

c. Range compression is accomplished by correlating of the phase corrected ISAR signal $\tilde{S}(p,k)$ with reference function complex conjugated of the transmitted Barker's code phase modulated signal $\mathbf{exp}[j\pi b((k-\hat{k}+1)\Delta T)]$, i.e.

$$\tilde{S}(\hat{p},\hat{k}) = \sum_{k=\hat{k}}^{\hat{k}+K-1} \tilde{S}(p,k)\mathbf{exp}[j\pi b((k-\hat{k}+1)\Delta T)] \quad (34)$$

where $p = \overline{1,N}$, $\hat{k} = \overline{1,L}$.

d. Azimuth compression and complex image extraction is accomplished by Fourier transformation of the range compressed ISAR data, i.e.

$$a_{ijk}(\hat{p},\hat{k}) = \sum_{p=\overline{1,N}} \tilde{S}(p,\hat{k})\mathbf{exp}\left\{j\left[\frac{2\pi}{N}\hat{p}p\right]\right\}. \quad (35)$$

Then the image of the target can be presented as

$$\mathrm{mod}\left[a_{ijk}(\hat{p},\hat{k})\right] = \left|\sum_{p=\overline{1,N}} \tilde{S}(p,\hat{k})\mathbf{exp}\left\{j\left[\frac{2\pi}{N}\hat{p}p\right]\right\}.\right| \quad (36)$$

Phase correction is performed by means of 2-D image entropy minimization. After range bin alignment, all the scatterers of the target remain into their initial range bins. In spite of this an unknown phase error is still left in the ISAR signals, which needs to be estimated.

4. Phase Correction Based on Entropy Minimization

If the image obtained by the procedure (20) and (36) is blurred an autofocus procedure has to be applied. It means to perform multiplication of the matrix $\hat{S}(k,p)$ by an exponential phase correction function $\exp[j\Phi(p)]$, i.e. $\tilde{S}(k,p) = \hat{S}(k,p).\exp[j\Phi(p)]$ which requires definition of $\Phi(p)$, i.e. to calculate the coefficients $a_2 \ldots a_m$ of the polynomials (17) and (30). The coefficients are calculated stagewise. At the first stage a_2 is calculated, at the second - a_3 and etc. The exact value of the coefficient a_m is computed iteratively, starting from $a_m = 0$ and incrementing by step $\Delta a_m = 0.01$ if the image quality gets better. If the image quality does not improve or gets worse go to computation of the next coefficient a_{m+1} or stop the procedure. In practice the quadratic term has a major impact on the phase correction process.

Let $\Phi_s(p)$ be the phase correction function, defined at the *s*-th stage. Then the autofocus phase correction is accomplished by multiplication of the demodulated ISAR signal, matrix $\hat{S}(k,p)$ with the exponential term $\exp(-j\Phi_s(p)$, i.e.

$$\widetilde{S}_s(k,p) = \hat{S}(k,p)\exp(-j\Phi_s(p)). \tag{37}$$

After current phase correction (16) and (33) image extraction by expressions (20) and (36), define the power normalized image as

$$\bar{I}_s(\hat{p},\hat{k}) = \frac{\left|a_{ijk}(\hat{k},\hat{p})\right|^2}{\sum_{p=0}^{N-1}\sum_{k=0}^{K-1}\left|a_{ijk}(\hat{k},\hat{p})\right|^2}. \tag{38}$$

Define the image cost function as entropy function of the normalized ISAR image

$$H_s = -\sum_{p=0}^{N-1}\sum_{k=0}^{K-1} a_{ijk}(\hat{k},\hat{p})\ln[a_{ijk}(\hat{k},\hat{p})]. \tag{39}$$

The procedure is repeated until the global minimum value of the entropy H_s is acquired. The estimate of the phase correction function corresponds to the minimum of the entropy image cost function, i.e.

$$\hat{\Phi}_s(p) = \arg\min_{\Phi}\left\{H[a_{ijk}(\hat{k},\hat{p})]\right\} \tag{40}$$

5. Numerical Experiment

Assume a target, flying helicopter, illuminated by 13 elements Barker's phase code modulated pulses with carrier frequency *f* = 10 GHz, number of range samples 86, number of emitted pulses 360, pulse repetition period 1ms. In Fig. 2 real and imaginary parts of ISAR signal are presented. In Fig. 3 unfocused and focused images are shown.

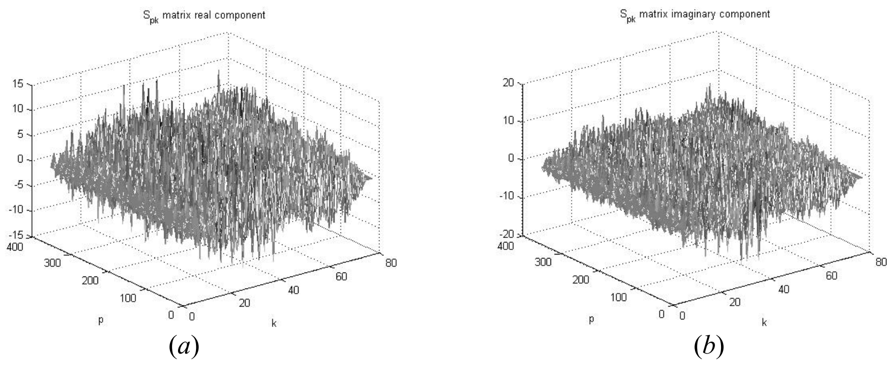
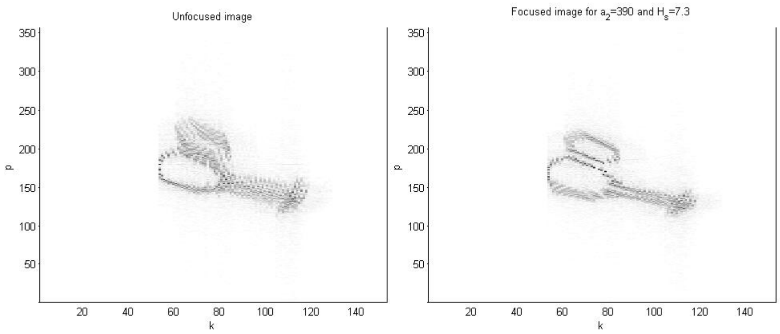


Fig. 2. ISAR signal: real part (*a*) and imaginary part (*b*) a projection of 3-D image function onto 2-D ISAR signal plane.



(*a*) (*b*)

Fig.3. Final images: unfocused image (*a*) and focused image (*b*), an inverse projection of 2-D ISAR signal plane into 2-D image function.

6. Conclusion

In this paper ISAR signal models based on LFM and PCM waveforms are defined. It is proven that signal formation is a projection operation of the image function onto 2-D signal plane and can be considered as direct space transform. Accordingly the image extraction is an inverse projection of 2-D signal plane into 2-D image function and can be interpreted as an inverse space transform. Moreover based on the projective operator derived the image reconstruction can also be interpreted as a phase compensation procedure, which compensates all ISAR signal phases induced by motion of the target.

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