MRW model of growth: foundation, developments, and empirical evidence

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1. Introduction

The economics of growth is one of the most popular fields of study in both theoretical and empirical economics. There are two main strands of models which try to find sources of long-run growth in the contemporary economies: the neoclassical growth model and endogenous model of growth (Romer 1986, Lucas 1988). Originally, the basic neoclassical Solow-Swan model suggests that the latter is a function of the nation's stock of labor and physical capital (Solow 1956, Swan 1956). Yet, in view of the unfavorable demographic conditions in the developed world it is not labor as a primary input but rather human capital, specifically education that is expected to create a growth friendly environment. That, in turn, determines the increasing research interest in the impact of education on the economic development.

With regard to that, this study discusses the neoclassical model of growth extended with human capital. It aims at presenting the theoretical foundation of the model of Mankiw, Romer and Weil (1992) henceforth MRW model, its extensions as well as empirical evidence built upon it. The structure of the paper is as follows. Section 2 focuses on the mathematical description of the MRW model whereas section 3 sheds light on the most significant developments. Some concluding remarks are presented in the last section (section 4).

2. Theoretical framework and mathematical description of the MRW model

The standard Solow-Swan model of growth is based on the aggregate production function of Cobb-Douglas type involving two production inputs:

$$Y(t) = A * K(t)^{\alpha} * L(t)^{1-\alpha} \quad (1)$$

It assumes a constant state of technology denoted by A, labor supply (L) growing at rate n, an exogenous savings rate (s) as well as a constant depreciation rate (δ). Mankiw, Romer and Weil et al. (1992) augment the model by adding human capital (H) as a third separate production input. The model keeps the assumption for a Cobb-Douglas aggregate production function with constant returns to scale as the next expression shows:

$$Y(t) = A(t) * K(t)^{\alpha} * H(t)^{\beta} * L(t)^{1-\alpha-\beta} \quad (2)$$

The following standard notations are used: Y is output, K denotes the stock of physical capital, H is the stock of human capital, L – the supply of labor, while A represents the level of technology; α and β measure the output elasticity with respect to physical and human capital, respectively. L and A are expected to grow exogenously at rates *n* and *g*:

$$L(t) = L(0) * e^{nt} \quad (3a)$$
$$A(t) = A(0) * e^{gt} \quad (3b)$$

Thus, it can be derived that physical and human capital expressed in effective units of labor evolves as follows:

$$\dot{k}_t = s_k * y_t - (n + g + \delta) * k_t \quad (4a)$$

$$\dot{h}_t = s_h * y_t - (n + g + \delta) * h_t \quad (4b),$$

In the above system of equations the small letters – $k = K/AL$, $h = H/AL$ and $y = Y/AL$ – denote quantities per an effective labor unit. sk and sh present the rate of accumulation of physical and human capital, respectively. Additionally, both types of capital depreciate at the same rate (δ). The existence of diminishing returns to capital implies that $\alpha + \beta < 1$. Under these initial conditions, the capital follows a convergence path to the steady state (k^*, h^*) given by the system of equations (5):

$$k^* = \left(\frac{s_k^{1-\beta} * s_h^{\beta}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}} \quad (5a)$$

$$h^* = \left(\frac{s_k^{\alpha} * s_h^{1-\alpha}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}} \quad (5b)$$

Substituting (5) into the production function (1) and taking logs we could express the equilibrium

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level of income per capita (y) in two alternative ways: either as a function of investments in human capital s_h (eq. 6a) or as a function of the human capital level h^* (eq. 6b).

$$\ln y(t) = lnA(0) + gt - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta) + \frac{\alpha}{1-\alpha-\beta}\ln(s_k) + \frac{\beta}{1-\alpha-\beta}\ln(s_h) \qquad (6a)$$

$$\ln y(t) = lnA(0) + gt - \frac{\alpha}{1-\alpha}\ln(n+g+\delta) + \frac{\alpha}{1-\alpha}\ln(s_k) + \frac{\beta}{1-\alpha}\ln(h^*) \qquad (6b)$$

Mankiw et al. (1992, c. 428) suggest that the form of the structural model built upon (6) should depend on the available data. If the time series correspond more closely to rate of investments in human capital, then (6a) is recommendable; otherwise, if a measure of the human capital stock is preferable, the regression model should resemble (6b).

The short-run dynamics that is the convergence of income per capita to its steady-state level is:

$$\ln(y_t) - \ln(y_0) = \left(1 - e^{-\lambda t}\right)\ln(y^*) - \left(1 - e^{-\lambda t}\right)\ln(y_0) \qquad (7).$$

It implies that the change of income per capita is a function of the determinants of both the ultimate steady state (y*) and the initial level (y0) of income per effective unit of labor. Substituting for the steady state (y*), the last equation becomes:

$$\ln(y_t) - \ln(y_0) = \left(1 - e^{-\lambda t}\right)\frac{\alpha}{1-\alpha-\beta}\ln(s_k) + \left(1 - e^{-\lambda t}\right)\frac{\beta}{1-\alpha-\beta}\ln(s_h) - \left(1 - e^{-\lambda t}\right)\frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta) - \left(1 - e^{-\lambda t}\right)\ln(y_0) \qquad (8)$$

The parameter λ measures the rate of convergence to the long-run equilibrium. It might be shown that if the share of each capital input is α = β = 1/3, and (n+g+δ) = 0.06, then the convergence rate would equal 0.02.[1] This is twice slower than the prediction of the basic model. Since in the Solow model β = 0, the faster convergence is implied (λ = 0.04).

In the neoclassical model long-run GDP increments would exist only if the population grows. GDP per capita is constant in the steady state. In case of a constant rate of growth of technological progress (g) as it is presumed in (3b), income per capita is expected to increase at the same rate (g) in the steady state. The main problem here is that no rationale for where this technological progress could come from has been given. If it is driven by innovation, then innovators cannot be rewarded, as output is already exhausted by payments to capital and labor. The model's augmentation by human capital shares some similarities with the basic model but, on the other hand, it solves some problems imposed by the latter. They are summarized as follows:

- There is still no long run growth of GDP per capita due to the presence of decreasing returns to capital accumulation.
- There is still no role for technological progress or the latter, if any, cannot be explained.
- Convergence to the steady-state is slower as human capital accumulation partly offsets the impact of decreasing returns to physical investments. It is equivalent to allowing for a broader capital share in the Solow model (α). The slower convergence rate is in conformity with the real-life pattern.
- The inclusion of human capital also magnifies the impact of the saving rate in physical capital on steady-state income (y*) since the elasticity of y* with respect to sk is (please, refer to eq. 6):

$$\frac{1}{1-\alpha-\beta} > \frac{1}{1-\alpha}.$$

On the empirical side, the MRW model is usually solved by a restricted regression equation which relates the GDP per capita or working age population to the inputs of production. The saving rate s_k is measured by the share of overall or business investments in GDP. The parameter s_h is calculated as the average percentage of the working-age population in secondary school over a long time period. Mankiw et al. relate the latter over the period 1960-1985 to GDP per working age person in 1985. The sum (g + δ) is set to 0.05 for all countries.

Alternatively, the equilibrium human capital stock (h^*) in (6b) might be well approximated by

[1] *This comes from the equation for the convergence rate λ = (n+g+δ)*(1-α-β)*

either the active population having completed a certain educational degree, i.e. at least upper secondary education (Neycheva 2013, 2016) or the average years of schooling available in the well-known Barro-Lee database (Barro-Lee, 2013).

3. Main extensions of the original MRW model and empirical findings

Being a popular structural model for evaluation of long-term growth, the model of MRW has been experiencing many developments. The latter fall into *two categories*: 1). modifications concerning the model's structure, and 2). usage of alternative approaches and methods to solve the model. The primary goal of both sorts of improvements is to achieve a higher degree of explanatory power of the MRW model since the coefficient of determination (adj. R2) for the advanced OECD countries was no larger than 0.30. The following lines review the most essential developments of the model.

Among the papers in the *first group*, a significant contribution has been brought by *Nonneman & Vanhoudt* (1996) who suggest further augmentation by explicitly including the endogenous accumulation of technological know-how. Their production function comprises m types of capital such as infrastructure, equipment, other physical capital, human capital, know-how, etc. Technological know-how, in the sense of blueprints for production processes and new products is considered a form of capital included in the production function as any other input.

The specific model which they solve assumes three production factors (m = 3): physical capital (k), human capital (h) and technological know-how (τ). The latter is approximated by the ratio of gross domestic expenditure on research and development to nominal GDP. In this way, the explanatory power of the model increases more than three times as adj.R^2 jump from 0.220 in case of human and physical capital only (m= 2) to 0.732 for the full specification (m = 3). Therefore, the authors claim that their model explains 80% of the variation in the cumulative growth rates between OECD countries compared to 65% in the original MRW study.

Knowles and Oven (1995) add health capital as, according to the broader view, an important aspect of the human capital quality. Using the life expectancy as an indicator of the national health care capital stock they succeeded in increasing the goodness-of-fit for the high income economies up to 0.71. The main finding is that health capital is more significant that educational capital for growth.

As it was mentioned above, the second strand of papers related to the Solow-Swan model augmented by human capital places an emphasis on the econometric issues. The hypothesis that all countries have identical production functions with the same parameters appeared to be too restrictive therefore the study of *Islam* (1995) is the first to relax that assumption. He retains the same 5% rate for the labor-augmenting technical progress plus physical capital depreciation across countries while allowing the aggregate production function to vary with respect to the productivity shift parameter. Using panel data he solves the model by dividing the whole time period to 5-year intervals.

A number of empirical papers follow Islam's estimation method. Among these is the work of *Easterly and Levine* (2001). Regional dummies as proxies of varying productivity levels and thus technological parameter A have been introduced in the production function. The study goes beyond the MRW model by investigating the link between economic policies and growth. Education, openness to trade, inflation, and government size appeared to be strongly linked to economic growth.

Lee, Pesararan and Smith (1997) continues the developments in this direction by allowing the countries to differ in level effects, growth effects and speed of convergence since there is a significant dispersion in the growth rates and speed of convergence. They derive a stochastic version of Solow model where the heterogeneous parameters were modeled in terms of random coefficients model and used exact maximum likelihood estimation. In addressing the problem of heterogeneity *Madala and Wu* (2000) apply an iterative Bayesian approach. They claim that they improve the results of Lee et al. (1997) whose method of estimation is not fully efficient in the presence of lagged dependent variables.

It is worth noting the contribution of *Durlauf and Johnson* (1995) as well. A regression tree has been used in order to allow the data to identify multiple data regimes and divide the countries into groups, which share a common statistical model. Then, Temple (1998) uses robust estimation. When removing Portugal and Turkey from the OECD sample, the fit in the regression decreased from 0.35 to 0.02. That demonstrates that the model has almost

no explanatory power for the most advanced economies.

Felipe and McCombie (2005) propose an alternative solution for improving upon the poor results for the OECD sample by relaxing the assumption of a common rate of technical progress. The latter may be determined from the dual of the production function and is likely to differ among countries. Once calculated, it might be included in the regression. The level of technology is evaluated by the following expression:

$$A(t) = B_0 w(t)^{1-\alpha} r(t)^{\alpha} \quad (9)$$

The growth rate of the wage rate is wt, rt is the change of the profit rate whereas α is capital's share in output. The power of the structural model built upon the proposition for different technology across OECD countries significantly increases as R^2 approaches 0.85.

Another measurement problem in the growth regressions has been addressed by the influential study of *Hanushek and Kimko* (2000). They point out that indicators of formal schooling such as primary- or secondary enrollment rate might accurately represent neither the relevant stock of human capital of the labor force nor the increments of the stock during periods of educational and demographic conditions. Therefore, they run a reduced form growth regression which links the average annual growth rate of real per capita GDP to a labor force quality indicator specifically cognitive skills of secondary school students measured by scores from standardized tests. The addition of the schooling quality to the quantity of human capital leads to a more than twice increase of R^2: to 0.73 from 0.33.

As it was already mentioned, the MRW model has been estimated primarily by cross-country regressions. A disadvantage of that approach is that it does not allow the differences in the growth patterns of the countries in the sample to be recognized. Few papers apply a time series analysis to a single country case. In a study on the Greek economy *Tsamadias and Prontzas* (2012) relate gross secondary enrollment ratio in a certain year to GDP growth over the next two years. Yet, it needs longer time to fully integrate secondary school students in the labor market.

The next problem deserving attention is that the MRW model does not distinguish between the different types of human capital. That might arise as an important specification problem if the different classes of human capital for e.g. the labor force with primary, secondary or tertiary education differ in productivity and thus impact on growth. With regard to that, *Neycheva* (2016) subdivides the national stock of human capital by differentiating between investments in secondary and in tertiary education. On the basis of reduced-form regressions the study presents estimates for the impact of educational levels on growth for three economies: Bulgaria, Czech Republic and Estonia. In Bulgaria human capital has either negative or non-significant impact on the long-run rate of growth of per capital income, while in Estonia higher education appears to be positively related to it. A plausible explanation for the differences in the regression outcomes is the higher degree of vertical qualification mismatch being found for the former.

4. Conclusion

The dominant paper of Mankiw, Romer and Weil (1992) sets a fundamental framework for estimating the sources of long-run growth in the modern market-based economies by introducing human capital as a separate production input in the Solow-Swan neoclassical model. Being the first of its kind, the model has been continuously improved and extended in order to increase the goodness-of-fit especially for the advanced countries. With regard to that this study aims to summarize the most important contributions related to the structural as well as econometric specification of the MRW model. Moreover, the critical review outlines some directions for further extensions and improvements such as: an inclusion of land and natural resources as a separate production input for countries which strongly rely on that factor of production; a disaggregation of the human capital stock and an assessment of the impact of its components; an improvement of the methods of estimation in order to ensure robustness of the outcome.

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